

Lab 2

Calculating Angles

1 Introduction

This document will serve as the specification (“specs”) for the program you are being asked to write. Be glad that none of you are programming for a living, since the specs you usually get are not nearly this nice!

Once you understand what needs to be done, your task is to create a MATLAB function which *implements* the specs you have been given.

Datasets and other files that you need for this assignment can be downloaded from:

<http://bigiron.atistar.net/~stepp/coding/>

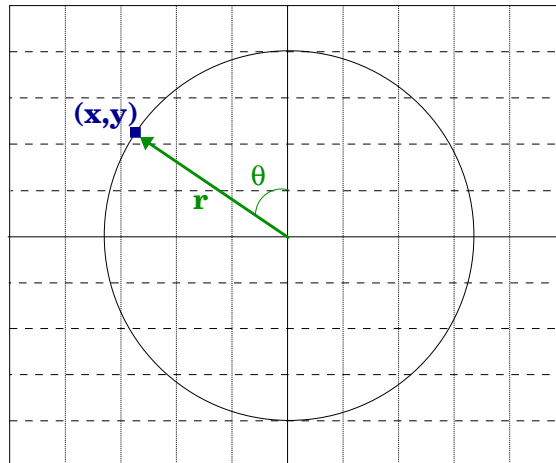
2 The Problem

An experimenter¹ has captured data of a subject drawing a circle on a table top. The data is recorded as motion in two perpendicular axes, we will call them x and y .

Problem: We have two sinusoidal time series which represent motion on a circle. What we are interested in, however, is the angle of each point around the circle.

Restriction: To make things fun, I’d like you to solve the problem *without* using *any* loops. This will be an exercise in taking advantage of MATLAB’s ability to do vector calculations quickly.

To make the angle in question clear, see the following diagram which shows both the (x, y) cartesian coordinate on a circle, and the (r, θ) polar coordinate.



¹This data is from Howie Zelaznik.

2.1 Program Specification

Write a function named `calc_circle_angles` with *minimally* the following function declaration:

```
function [r theta] = calc_circle_angles( xy_data )
```

This function takes a single parameter which is the (x, y) circle data to be transformed into (r, θ) data.

Note that I say *minimally* above, your implementation may require additional parameters.

The output parameters, `r` and `theta`, are vectors the same length as `xy_data` containing the r and θ polar polar coordinates respectively.

2.2 Implementation

Your function should be saved in a file named `calc_circle_angles.m`.

The way your implementation solves the problem is up to you, but if you need to get started just send me an email.

Once you have run your function, and are happy with your output, send your `calc_circle_angles.m` to me. I'll give you constructive comments and hints on ways to make it better.

3 Vector Math

It will be very useful to know how to calculate the angle between two vectors. So as to assume no knowledge of vector calculations, here is a small tutorial.

3.1 Vectors themselves

Vectors are nothing more than lists of numbers. $(1, 5, 7)$, $(1, 6, 7, 5, 3, 2, 3, 1)$, (9) . The number of numbers in each list (each vector), is called the *dimension* of the vector².

The numbers *inside* the list are named scalars. Scalars can be almost anything that you can add or multiply. Integers, real numbers, complex numbers. . . these are all scalars.

3.2 Dot products

There exists an operation between two vectors, similar to multiplication (product), which produces a scalar. This multiplication of two real number vectors, for instance, will result in a single real number.

If we have two vectors p and q , we show this operation using a dot: $p \cdot q$. Because of the dot, we say this is a “dot product”.

Dot products are easy to compute. If $p = (4, 5, 6)$ and $q = (9, 2, 1)$ then:

$$p \cdot q = 4 \times 9 + 5 \times 2 + 6 \times 1 = 52$$

As you can see, simply take each element, multiply it by the corresponding element in the other vector, and add everything together.

Remember the “Sum of Squares” and “Sum of Products” from stats? These are actually dot products!

3.3 Vector lengths

It turns out (think pythagorean theorem), that the dot product of a vector with itself is related to the length of the vector. Using p from above, the length of p , or $\|p\| = \sqrt{p \cdot p}$.

²Ok, so this is a little simplified, but basically that's it.

3.4 Angles between vectors

Now we have the tools needed to calculate the angle between two vectors. I will spare you the derivation, and just let you know that there is an equation for this angle.

Again, given vectors p and q , the angle between them is given by:

$$\theta = \arccos\left(\frac{p \cdot q}{\|p\|\|q\|}\right)$$

A small hint: this calculation is much easier if one of the vectors has a length of 1.

Also, a fun stats note: Do you recognize the correlation coefficient in this equation?