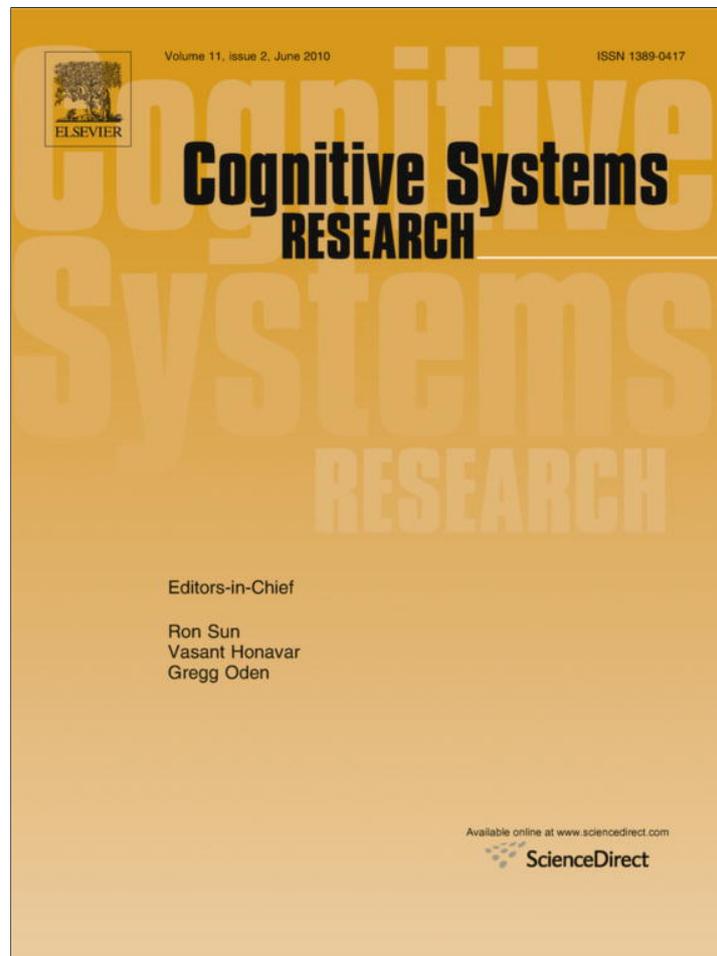


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# On strong anticipation

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Received 1 October 2008; accepted 22 March 2009  
 Available online 27 May 2009

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## Abstract

We examine Dubois's [Dubois, D., 2003. Mathematical foundations of discrete and functional systems with strong and weak anticipations. *Lecture Notes in Computer Science 2684*, 110–132.] distinction between weak anticipation and strong anticipation. Anticipation is weak if it arises from a model of the system via internal simulations. Anticipation is strong if it arises from the system itself via lawful regularities embedded in the system's ordinary mode of functioning. The assumption of weak anticipation dominates cognitive science and neuroscience and in particular the study of perception and action. The assumption of strong anticipation, however, seems to be required by anticipation's ubiquity. It is, for example, characteristic of homeostatic processes at the level of the organism, organs, and cells. We develop the formal distinction between strong and weak anticipation by elaboration of anticipating synchronization, a phenomenon arising from time delays in appropriately coupled dynamical systems. The elaboration is conducted in respect to (a) strictly physical systems, (b) the defining features of circadian rhythms, often viewed as paradigmatic of biological behavior based in internal models, (c) Pavlovian learning, and (d) forward models in motor control. We identify the common thread of strongly anticipatory systems and argue for its significance in furthering understanding of notions such as "internal", "model" and "prediction".

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*Keywords:* Anticipation; Synchronization; Delayed feedback; Homeostasis

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## 1. Introduction

The capabilities of living things are very much bound up with the notion of anticipation. Memory is as much prospective (e.g., remembering what to do when) as it is retrospective (e.g., remembering what happened when). Actions are as much proactive as retroactive, and probably more so, with perception often identified as the immediate means of controlling actions in a forward-looking, prospective manner. Behaving felicitously in everyday cluttered environ-

ments depends on an ability to apprehend what behaviors are possible and in what ways they might be performable.

In current debates about how to approach cognitive systems, anticipation plays a particularly pivotal role. For those inclined to the traditional view of mediation of behavior by representations, anticipation would seem to be an especially "representation-hungry" problem (Clark, 1997; Clark & Grush, 1999), that is, one that cannot possibly be manifest without special internal states. To so behave as to be ahead of forthcoming events must presumably require a source of information (i.e., representations) that goes beyond the information currently available to perceptual systems. For those who are not inclined to appeal to mediating representations, the foregoing presumption must necessarily be wrong (Keijzer, 2001). There must be more to the organism and its environment vis-à-vis information and dynamics

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than has so far been acknowledged: “Perhaps behavior is not anticipatory at all, but a result from immediate organism–environment couplings p. 192 (Keijzer, 2001)”.

Although much has been written about the relation between representations and cognition, relatively little has been written about the relation between couplings and cognition (for notable exceptions see) (Beer, 1995, chap. 5, Beer, 2009, Kelso, 1995; Warren, 2006). In this article we present an exemplary case in point for how couplings can yield behavior that would commonly be labeled “anticipatory”. We make modest steps toward an affirmative answer to the question: Can the claim of “internal representations mediate anticipatory behavior” be reformulated as “organism–environment couplings engender reactive behavior qua anticipatory behavior”?

## 2. The notion of anticipation

Anticipation as an English word has a wide variety of senses and connotations. If I think it will rain tomorrow, I might put my umbrella by the door in anticipation of needing it. I may also say that I intercept a projectile by anticipating its future position. In the following text, *anticipation* is meant as a technical term implying the latter. Specifically, we mean something closer to the original Latin meaning of ‘to take before’, especially ‘to follow a path before’.

For our purposes, “to anticipate” is precisely for one aspect (or state) of a system to maintain a negative phase relationship with some other aspect, possibly within a separate system. Such a definition encompasses situations where some part of a system follows the future trajectory of another. Exactly how this is possible is the problem at hand, with application to issues such as the prospective control of action and predictive homeostatic regulation.

## 3. How can one system anticipate another?

In an influential work Rosen (1985) argued that for purposes of anticipation a system must contain a predictive model of itself and its inputs, which allows it to change adaptively at instant  $t$  in accord with the model’s predictions about  $t + \tau$ . Craik (1943) had underscored the significance of predictive models some decades earlier:

If the organism carries a “small scale model” of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize knowledge of past events in dealing with the present and future, and in every way react in a much fuller, safer, and more competent manner to the emergencies which face it.

Craik saw the internal model as having a similar “relation-structure” to the thing modeled (in the sense of “working in the same way”) and as being implemented in terms of

symbols and symbol manipulation. The profound advances in symbol manipulating machines since Craik’s day have boosted the feasibility of Craik’s argument and the willingness of theorists to propose internal predictive models.

For an organism to be predictive, to be able to anticipate, implies two things: (a) the existence of constraints (on the states of the environment and on the states of the organism’s body) and (b) sensitivity, on the organism’s part, to the existing constraints. By definition a constraint on a thing or process means that the thing or process cannot exhibit all of its potential variety. Certain aspects will remain unchanged, or invariant, while other aspects are free to vary. Invariants imply constraints.

Given that constraints provide the necessary grounding for the ability to predict, a particular challenge facing any scientific attempt to address the question of “How one system can anticipate another” is that of identifying the constraints relevant to the specific systems covered by the question. There are two very different ways of meeting the challenge. Whereas one way conforms to the intuitions of Rosen (1985), Craik (1943), the other way does not. We will exemplify the two ways through the so-called outfielder problem.

### 3.1. An analytic solution to the outfielder problem

In the outfield, the baseball player responds to a well-struck ball by moving to the place where the ball is heading and at a pace that will get the player there at the right time. The constraints on the flight of the struck ball are those captured by the laws of mechanics. As depicted in Fig. 1, physics will ensure that, given the initial conditions, the ball will land at a point  $d$  meters from the location at which its flight originated at a time  $t$  seconds after it was struck. Not yet making any implication about implementation within an actual outfielder, the point and time of landing can be predicted by encoding the state of the ball into numbers that correspond to the ball’s physical variables. Given the assigned numbers, Newtonian mechanics provides the tools to obtain the numbers  $d$  and  $t$ . For one who comprehends

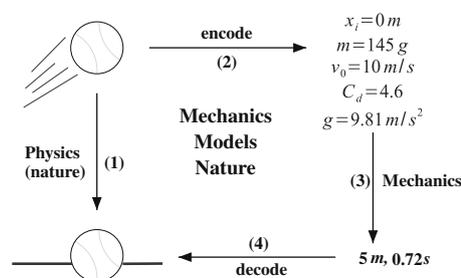


Fig. 1. The modeling relation describes what must happen for one system to model another. A ball following a ballistic trajectory, then landing, is controlled by the physics of the world. It may be modeled by mechanics first by encoding the initial state of the world into numbers, running through the mathematics, then decoding the answer into a physical state of affairs.

Newtonian mechanics, the numbers  $d$  and  $t$  are decoded into the meanings “the ball will land there” and “the ball will land then”, respectively.

The predictive process as diagrammed in Fig. 1 will be recognizable as Rosen’s (1991, 2000) modeling relation. This relation makes explicit what is required for one system to be a model of another. The outfielder may be anticipatory by virtue of modeling the ball-in-flight system that the outfielder is anticipating. The parts of the diagram labeled 2–4 are those that must be internalized implicitly by the outfielder. Recognition of the latter brings to the forefront a key feature of Fig. 1: it involves *two separate systems*, namely, the ball-in-flight (labeled 1) and the fielder (labeled 2–4). Within this example the ball-in-flight is, for all intents and purposes, the fielder’s environment. Again, this very numeric example is not meant to literally be the “internal model” as it is often spoken of, but it does share the same structure. This is exactly Rosen’s motivation for developing the modeling relation.

### 3.2. A non-analytic solution to the outfielder problem

Optical Acceleration Cancellation (OAC) and the Linear Optical Trajectory (LOT) are two non-analytic, non-model-based,<sup>1</sup> strategies for solving the outfielder problem (Michaels & Zaal, 2002). The two hypothesized strategies are schematized in Fig. 2. Both can trace their lineage to Chapman (1968). The two hypotheses are competitors, and each has been extended and modified in various ways, but they share something crucial in common.

The OAC strategy (McLeod & Dienes, 1996; Oudejans, Michaels, Bakker, & Dolné, 1996) accommodates all orientations of the outfielder to the ball, including the difficult case of catching a fly ball that is in the outfielder’s sagittal plane. The significant optical variable is hypothesized to be the vertical optical acceleration of the ball’s projection onto the image plane. The outfielder so behaves as to nullify this optical quantity (for simulations see) (Kim & Turvey, 1998). To elaborate, Fig. 2a suggests that

<sup>1</sup> Within the confines of the current discussion, we might test whether a system should be considered using a model by placing it into the four place relationship formulated by Giere (2004), “ $S$  uses  $X$  to represent  $W$  for the purposes  $P$ .” As an example, we might say, “The outfielder ( $S$ ) uses Newtonian mechanics ( $X$ ) to represent the flight of the ball ( $W$ ) for the purposes of predicting its future position ( $P$ ).” The natural way to write such a sentence for LOT/OAC is, “the outfielder ( $S$ ) uses a particular invariant property ( $X$ ) to represent when the outfielder’s heading is correct ( $W$ ) for the purposes of arriving when and where the ball lands ( $P$ ).” In this sentence, the relationship between  $X$  and  $W$  is not consistent with representation. It suffices to point out that, while representation is anti-symmetric ( $A$  represents  $B \Rightarrow B$  does not represent  $A$ ), the relation  $\sim$  between  $X$  and  $W$  in the preceding sentence is not ( $X \sim W \Leftrightarrow W \sim X$ ). The particular invariant property and the outfielder’s heading are logically entangled in a way that violates the symmetry properties of a representational relation. Therefore, we conclude the sentence is false, and that this characterization of LOT/OAC is not an instance of model use. For other discussions of models (see Eykhoff, 1974; Forster, 2000; Morgan, 2001; Webb, 2001; Suárez, 2003; Seidewitz, 2003; Frigg, 2006).

catching a fly ball is a matter of adjusting one’s locomotion to keep the vertical velocity of the ball’s projection on the image plane constant (and not a matter of predicting and then running to where the ball will fall). The LOT strategy (McBeath, Shaffer, & Kaiser, 1995) depicted in Fig. 2b is restricted to cases in which the ball’s flight deviates from the sagittal plane, that is, when the ball has horizontal velocity with respect to the outfielder. By this strategy, the ball will be caught if the would-be catcher’s locomotion continuously linearizes the ball’s optical trajectory.

Both strategies rely on a common step. The outfielder is tied to the ball by the imposition of a relational property. When the proper organization of ball and outfielder is in place, the ball and outfielder become a *single system* with new dynamics. The latter aspect is a departure from the analytic method, in which the outfielder is in the role of observer-with-calculator only. For both strategies, where and when the fielder intercepts the ball are not predicted by a model, but entailed by the natural unfolding of this new single system. p. 173 Michaels and Zaal (2002) suggest the following: “Getting to the right place at the right time is not solved by prediction, but by continuously adapting the action to information.”

## 4. Weak and strong anticipation

The analytic and non-analytic methods characterized through the outfielder problem are two fundamentally different answers to the same question. The analytic method predicts an explicitly referenced future in the sense that the outfielder consults a model that provides an answer to the questions of where and when. In contrast, by causing the agent to become a part of the system that the agent is anticipating, the non-analytic method takes advantage of existing information about the future. This latter case avoids explicit reference to the future, which is what makes anticipatory acts seem so difficult in the first place.

The two cases have been recognized as fundamentally different concepts deserving of distinct labels p. 447 (Dubois, 2001).

[t]he systems described... deal with what is called “strong anticipation”, because the current states of these... systems are computed from past and/or present states but also from potential future states, anticipated from these systems themselves and not from models of these systems. Systems which compute their current states from potential future states, anticipated from models of these systems, deal with what is called “weak anticipation”

In short, prediction of the future given a model is labeled *weak anticipation* and prediction of the future not given a model—and relying instead on systemic lawfulness—is labeled *strong anticipation*. Accordingly,

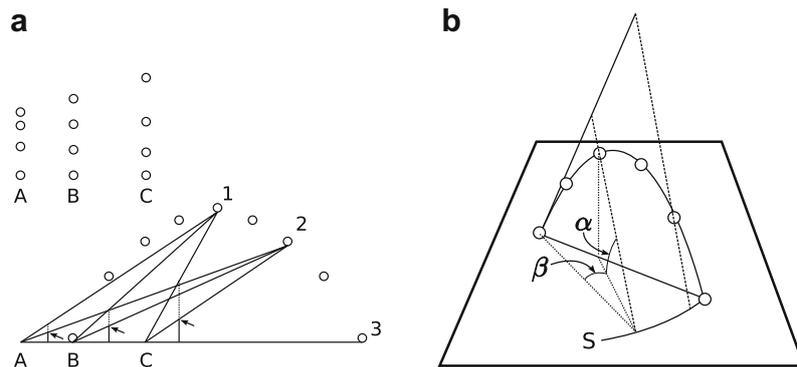


Fig. 2. (Left). So move as to keep the ball's optical vertical acceleration constant. A ball is depicted traveling a parabolic path from right to left with its position shown at equal temporal intervals. The ball is viewable from positions A, B, and C, where B is the location of the ball's landing point. Lines connect three positions of the ball in flight (1, 2, and 3) to the viewing positions (A, B, and C). Vertical lines, starting on the line to 1 and ending on the line to 3, identify image planes. Arrows indicate where the line to two intersects the image planes with a dashed segment above and a solid segment below each arrow. Relative to viewing positions A, B and C, dashed < solid for A, dashed = solid for B, and dashed > solid for C, meaning that, in the image plane, the ball is decelerating, of constant velocity, and accelerating, respectively. (The overhead array of points expresses the successive ball locations viewed from A, B, and C.) In summary, the ball will arrive at the eye if its optical image (the image on the plane) has zero acceleration (Adapted from Michaels and Zaal (2002).) (Right). So move as to linearize the ball's optical trajectory. Applies to the case when the ball's flight path is not in the catcher's sagittal plane. In the figure, the catcher starts at S. The ball's optical trajectory will be linear when the catcher runs on a path (identified by the arrow) such that tangents of the vertical angle  $\alpha$  and the horizontal angle  $\beta$  change proportionally. (Adapted from McBeath et al. (1995).)

anticipatory systems that exhibit strong anticipation can be referred to as strongly anticipatory systems in contrast to weakly anticipatory systems.<sup>2</sup>

### 5. Strong anticipation in a non-biological system

Viewed as strictly physical systems, the outfielder is to the ball as receiver is to transmitter, driven is to driver, and slave is to master. A physical example of a slave system coupled to a master system provides insight into the "proper organization" that yields a strongly anticipatory system.

Fig. 3a depicts a linkage between two external-cavity diode lasers, one in the role of master (transmitter, driver) and one in the role of slave (receiver, driven). Such lasers are known to have a chaotic regime expressible by the Ikeda Map (Ikeda, Daido, & Akimoto, 1980). When coupled in the right way, the dynamics of two such lasers exhibit what is called *anticipating chaotic synchronization* (Voss, 2000, 2001). Fig. 3a depicts a physical arrangement that implements the proper coupling (Sivaprakasam, Shahverdiev, Spencer, & Shore, 2001) as defined by Eqs. (1) and (2).

$$\frac{\partial x(t)}{\partial t} = -\alpha x(t) - \beta \sin x(t - \tau) \quad (1)$$

$$\frac{\partial y(t)}{\partial t} = -\alpha y(t) - \beta \sin x(t) \quad (2)$$

<sup>2</sup> A classic exemplar of weak anticipation is the Kalman Filter (Kalman, 1960; Maybeck & Siouris, 1980), which allows optimal estimation of discrete-time linear systems (although extensions do exist (Julier & Uhlmann, 1997)). The filter employs a predictor-corrector organization in which the predictor portion of the filter contains a model of the target process that is tuned by the corrector portion. The fact that the Kalman Filter requires an explicit model of the process to be estimated places it squarely into the category of weak anticipation (for example applications see) (Wu, Rink, Caelli, & Gourishankar, 1989; Riley, Ude, & Atkeson, 2000; Dorfmueller-Ulhaas, 2003).

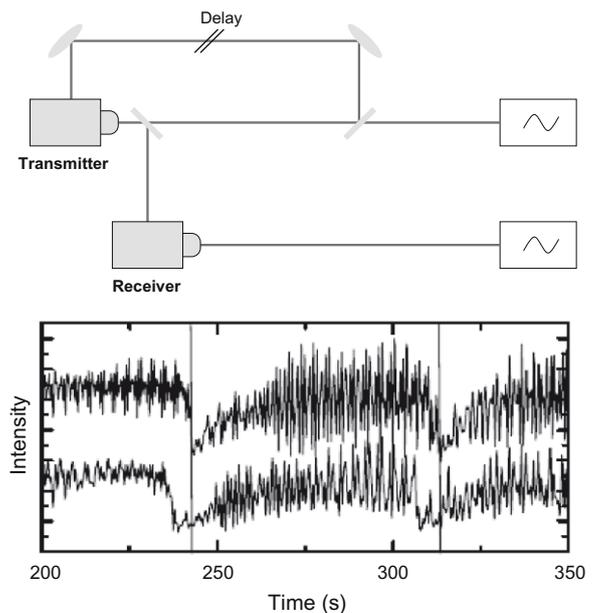


Fig. 3. Anticipating synchronization in laser systems. (Top panel) A schematic of the organization required for anticipating synchronization in a system made up of two coupled external-cavity diode lasers. The transmitter laser takes a delayed feedback input from itself. The receiver takes an input from the transmitter at negligible delay. (Bottom panel) Measured beam intensity over time, showing phase lead by the receiver laser (lower line) with respect to the transmitter laser (higher line). (From Sivaprakasam et al. (2001). Reprinted with permission from APS.)

A simulation of the system given by Eqs. (1) and (2) is shown in Fig. 4 and an example of actual data from Sivaprakasam et al. (2001) is shown in Fig. 3b. As is evident from inspection of Figs. 4 and 3b, the receiver ( $y$ ) is anticipating the state of the transmitter ( $x$ ), a potentially striking fact given the unidirectional coupling from the transmitter to receiver.

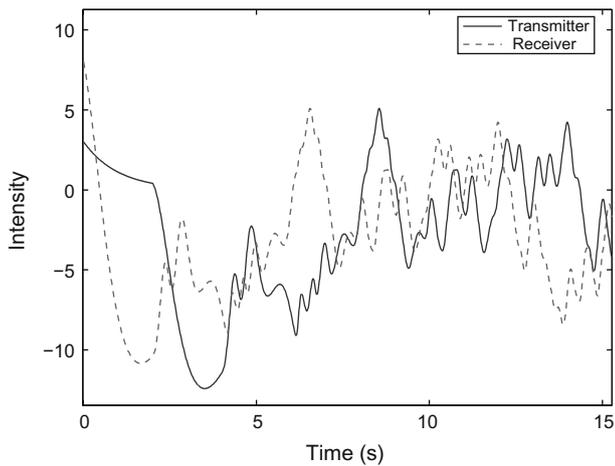


Fig. 4. Anticipating chaotic synchronization. Simulation of Eqs. (1) and (2) with initial conditions  $x(0) = 0$  and  $y(0) = 8$ ; and parameters  $\alpha = -1$ ,  $\beta = 20$ ,  $\tau = 2$ . The dashed line corresponding to the receiver ( $y$ ) is phase shifted to the left with respect to the solid line corresponding to the master ( $x$ ).

### 6. When $\tau$ inheres in the slave rather than the master

The Ikeda systems giving rise to the dynamics expressed in Figs. 3 and 4 belong to a category of systems wherein delayed feedback is present in the master system but is absent in the slave system. Systems for which delayed feedback characterizes the slave system and not the master system constitute another category that is potentially more encompassing of anticipating biological synchronization. Delays between initiation of processes and the adaptive manifestation of their effects are multiple in the physiology and behavior of organisms. Metabolic, genetic and motor control processes, among many others, are marked by delays. To deal with these delays, organisms must operate in an anticipatory fashion.

Voss (2000, 2001), Ciszak, Marino, Toral, and Balle (2004) have identified a general framework for anticipation by slave systems with delays:

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{y} &= f(y) + k(x - y_\tau) \end{aligned} \quad (3)$$

The term  $y_\tau$  identifies a past state of  $y$ , namely  $y(t - \tau)$ . This particular arrangement is often termed *delay-coupling*.

Eq. (3), where both systems have dynamics specified by  $f$ , is the typical way of formulating anticipating synchronization. This formulation, however, does not need to be so restrictive. It is the case that the dynamics of  $x$  and the dynamics of  $y$  need not be the same. It would be more fitting to show two different possible dynamics,  $f$  and  $g$ .

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{y} &= g(y) + k(x - y_\tau) \end{aligned} \quad (4)$$

### 6.1. Example systems

There are a great many choices for  $f$  and  $g$  in Eq. (4) which result in anticipating synchronization. Mathematics

and biology provide helpful examples of the variety of these dynamics. The following examples, however, are not strictly necessary for the understanding of what follows them. The reader has the option of skipping to Section 6.2 directly.

#### 6.1.1. The Rössler system

Originally developed to facilitate the study of the more complicated Lorenz system, the Rössler system (Rössler, 1976) has proven its usefulness, theoretically and practically (e.g., chemical equilibria) as the most elementary geometric construction of chaos in continuous systems (Peitgen, Jürgens, & Saupe, 1992). Analogous to Eqs. (1) and (2), it is possible to couple two Rössler systems in such a way that a unidirectionally driven slave system anticipates its master. The coupled Rössler system is given by the following equations, with  $x$  designating the master system and  $y$  designating the slave system.

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + ax_2 \\ \dot{x}_3 &= b + x_3(x_1 - c) \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{y}_1 &= -y_2 - y_3 + k(x_1 - y_{1,\tau}) \\ \dot{y}_2 &= y_1 + ay_2 \\ \dot{y}_3 &= b + y_3(y_1 - c) \end{aligned} \quad (6)$$

The two notable features of Eqs. (5) and (6) are (a) the completely autonomous nature of the master system and (b) the embedding of the delayed feedback term within the slaved system. A simulation of anticipating synchronization of Rössler master and slave systems is shown in Fig. 5.

#### 6.1.2. Excitable media

The notion of an excitable medium is a generalization of the feature of organisms and their components to respond strongly to an imposed, relatively weak stimulus (Winfree, 1987). Roughly, an excitable medium is a continuous

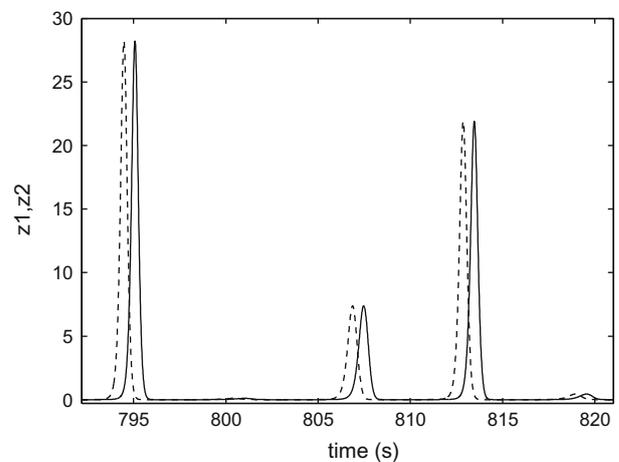


Fig. 5. Simulation of the Rössler system. Simulation of Eqs. (5) and (6) with parameters  $a = b = 0.1$ ,  $c = 14$ ,  $k = 1$ ,  $\tau = 0.6$ . State variables  $x_3$  (solid) and  $y_3$  (dashed) subsequent to transient are shown.

dynamical system with elementary parts that typically respond to a given influx of energy with a pulse or spike when the energy surpasses a threshold value. The energy in question may be noise or any external perturbation that induces a deviation from equilibrium. Local coupling suffices to propagate the excitation of one segment to its neighbors. Two examples of excitable systems are Adler's equation and the FitzHugh–Nagumo model of the neuron membrane potential.

Adler's equation is an abstract system commonly used to study the dynamics of excitable behavior in general. Again, two such systems may be coupled unidirectionally from master to slave, such that the slave anticipates the master. The specific coupling is shown in Eq. (7):

$$\begin{aligned} \dot{x} &= \mu - \cos x + I(t) \\ \dot{y} &= \mu - \cos y + I(t) + k(x - y_\tau) \end{aligned} \quad (7)$$

Because the coupled systems are meant to be excitable, a new term  $I(t)$  has been introduced as a way to provide the necessary excitatory stimulus. A small part of the simulation of anticipating synchronization in the coupled Adler systems is shown in Fig. 6.

The FitzHugh–Nagumo system addresses a more specific instance of an excitable system. It is a simplification of the Hodgkin–Huxley system of equations developed for the precise purpose of capturing spike generation in the giant squid axon. Coupled in the now usual way, two such FitzHugh–Nagumo systems show anticipating synchronization (Torralba, Masoller, Mirasso, Cizak, & Calvo, 2003). The equations and coupling in question are:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 - \frac{x_1^3}{3} + I(t) \\ \dot{x}_2 &= \epsilon(\alpha - x_1) \\ \dot{y}_1 &= y_2 + y_1 - \frac{y_1^3}{3} + I(t) + k(x_1 - y_{1,\tau}) \\ \dot{y}_2 &= \epsilon(\alpha - y_1) \end{aligned} \quad (8)$$

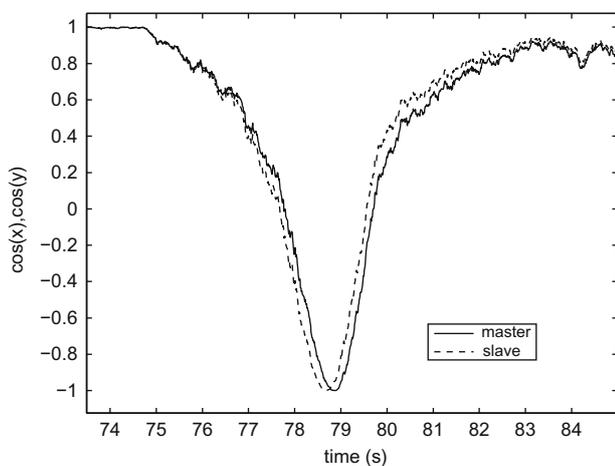


Fig. 6. Simulation of the Adler system. Simulation of Eq. (7) with parameters:  $\mu = 0.95$ ,  $k = 0.1$ ,  $\tau = 4$ . Only dynamics subsequent to transient are shown.

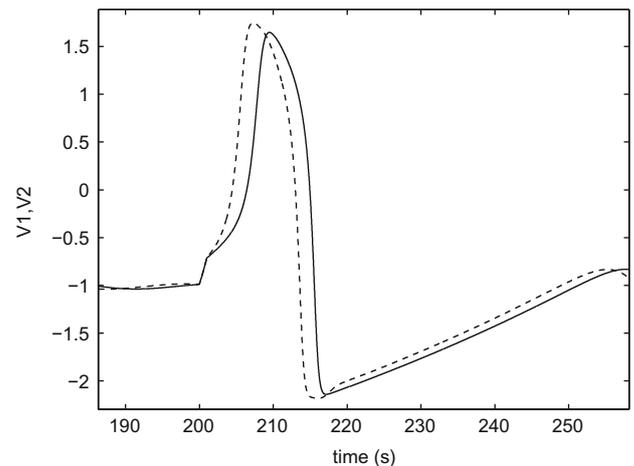


Fig. 7. Simulation of the FitzHugh–Nagumo system. Simulation of Eq. (8) with parameters  $\epsilon = 0.09$ ,  $a = -1.01$ ,  $p = 0.4$ ,  $k = 0.1$ ,  $\tau = 4$ . Only dynamics subsequent to transient are shown.

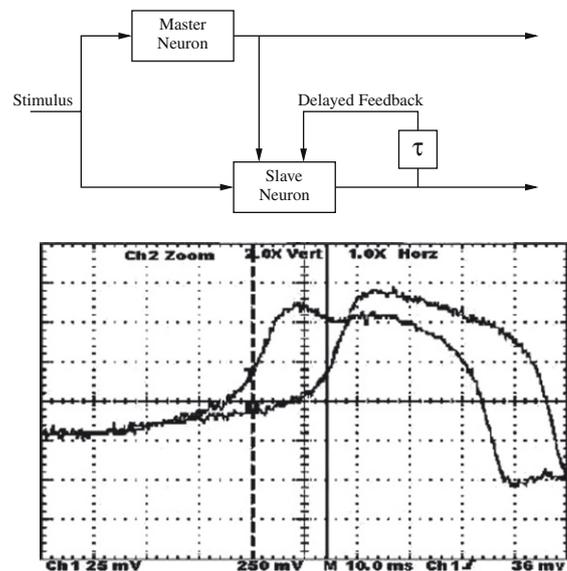


Fig. 8. Anticipating synchronization in neuronal systems. (Top panel) Schematic of the organization required for anticipating synchronization in a system made up of two coupled neurons. Both neurons receive input from the same external stimulus. The slave neuron additionally receives input from the master neuron and itself via delayed feedback. (Bottom Panel) Measurement of the actual neuron membrane potentials, showing anticipation by the slave neuron. (From Cizak, M., Marino, F., Toral, R. & Balle, S. (2004). Dynamical mechanism of anticipating synchronization in excitable systems. *Physical Review Letter*, APS, 93, 114102. Reprinted with permission from APS.)

Fig. 7 provides a simulation and Fig. 8 provides a schematic of coupled model neurons together with an example of an anticipated spike as follows from Eq. (8).

### 6.2. Regions of anticipating synchrony in the parameter space (coupling strength, delay)

As seen in Eq. (3), there are two parameters shaping the dynamics of the unified master–slave system, namely, the

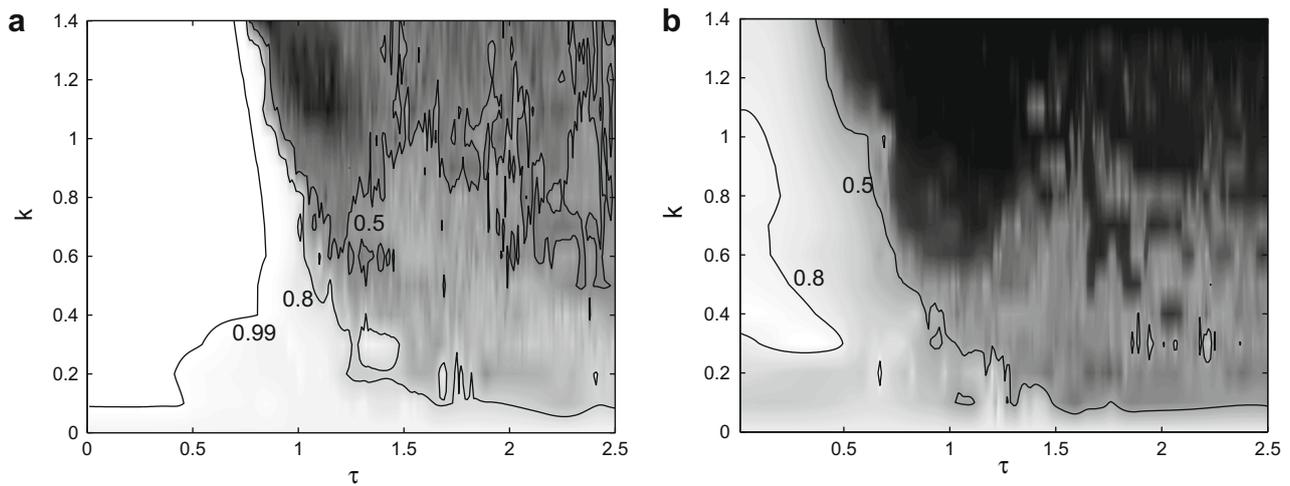


Fig. 9. Regions of anticipation in coupling-delay parameter space. By incrementally shifting the slave time series relative to the master, it is possible to obtain a measure of anticipation by calculating the maximum correlation of all shift increments. If some amount of positive shift produces a high correlation, the slave is anticipating well. (Left) master and slave parameters:  $a = b = 0.1, c = 14$ . (Right) master:  $a = b = 0.1, c = 14$ ; slave:  $a = b = 0.2, c = 5.7$ .

time delay  $\tau$  and the coupling strength  $k$ . There are, of course, other parameters present in specific master and slave systems (e.g.,  $a$ ,  $b$  and  $c$  in the coupled Rössler Eqs. (5) and (6)). Although  $\tau$  and  $k$  are the most prominent, all parameters affect the degree of anticipating synchronization, with any mismatch between those of master and those of slave rendering the anticipation less reliable. Panels (a) and (b) of Fig. 9 describe the  $(\tau, k)$  parameter space for the coupled Rössler system. Respectively, they reveal the strength of anticipating synchronization within regions of the parameter space for cases of matched and unmatched master–slave parameters ( $a$ ,  $b$  and  $c$ ). It is greater for the matched than the unmatched, though it should be noted that either case provides a large area of anticipation.

### 6.3. The mechanism of delay-coupling

In the absence of a delayed feedback term in the above systems, anticipating synchronization does not occur. It would seem that delay is a necessary condition. Delayed feedback in the slaved systems is the *sine qua non* of the strong anticipation evident in the coupled systems of the previous sections. The positive, productive status of delayed feedback cannot be overemphasized. A major motivation for pursuing weak anticipation (that is, internal predictive models) is the assumption that feedback delays are threats to successful control. Their very presence is regarded as an unfortunate feature of biological and robotic systems, one that must be compensated for if such systems are to behave felicitously (see Section 10) (Kawato, 1999).

The stabilizing effect of delayed feedback has been well investigated (Pyragas, 1992, 1998; Kittel, Parisi, & Pyragas, 1995). In a typical case of delay induced stability, the delayed system is synchronized with the non-delayed version of itself, i.e., a coupling arrangement such as  $k(x - x_\tau)$ . The usual effect is the stabilization of a limit cycle

with period equal to the delay. For anticipating synchronization, this method is applied across the master–slave boundary, instead of remaining within a single system.<sup>3</sup>

The effect of the coupling term  $k(x - y_\tau)$  is to minimize the difference between the state of  $x$  at the current time, and the state of  $y$  at a past time. If this difference is successfully minimized, then the difference between the present state of  $y$  and future state of  $x$  is also minimized. The effect of this minimization is the synchronization of  $y$  with the future of  $x$ .

As is apparent in Fig. 9, increasing delay eventually leads to the destruction of anticipating synchronization. We can see from the general formula and preceding examples that the coupling term affects, or is applied to,  $\dot{y}(t)$ . The coupling term, however, seeks to synchronize  $x(t)$  and  $y(t - \tau)$ , neither of which it is affecting directly.

For values of  $\tau$  close to 0, the term affects what it needs to directly and (non-anticipating) synchronization is easily reached. As  $\tau$  increases, the ability to affect  $y(t - \tau)$  in the correct direction by changing  $\dot{y}(t)$  plays heavily into the ability to achieve anticipating synchronization. The change required at a particular time must be highly correlated to the change required over the span of  $\tau$ . At higher values of  $\tau$ ,  $y(t)$  and  $y(t - \tau)$  may become uncorrelated, at which point anticipating synchronization is expected to break down.

### 6.4. Fit of slave to master

In the example systems of Section 6.1, the master and slave dynamics follow the form of Eq. (3), master and slave

<sup>3</sup> In this respect, anticipating synchronization is a straightforward use of standard concepts in control theory, and it may be analyzed as such (Blakely, Pruitt, & Corron, 2008). The issues at hand, however, are the implications of the phenomenon's existence and its connection to the broader concept of strong anticipation.

dynamics being the same, rather than the more general form of Eq. (4), master and slave dynamics being different. An obvious question is whether this feature is necessary. A simple simulation will serve as a counterexample to this necessity.

Consider a higher dimensional, chaotic system exemplified by the Rössler system in Eq. (5), and a lower dimensional system exemplified by a simple harmonic oscillator.

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -ky_1 \end{aligned} \quad (9)$$

Placing these two systems in the arrangement of Eq. (4), we see in Fig. 10 that the 2-dimensional harmonic oscillator (g) successfully falls into anticipating synchronization with the 3-dimensional Rössler system (f). In truth, by coupling the two systems it no longer makes sense to speak of

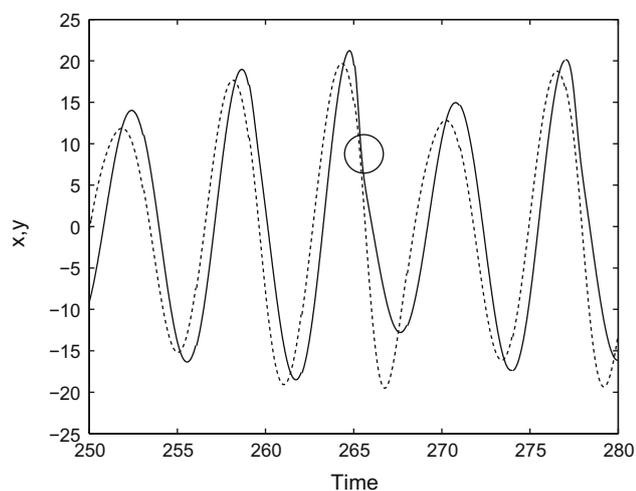


Fig. 10. Chaotic oscillator driving a simple harmonic oscillator. Rössler parameters:  $a = b = 0.1$ ,  $c = 14$ . Harmonic oscillator parameters:  $k = 1$ ,  $\tau = 0.6$ . The circle inscribes a point at which the chaotic master changes trajectory abruptly, with consequences for the slave (see Section 11).

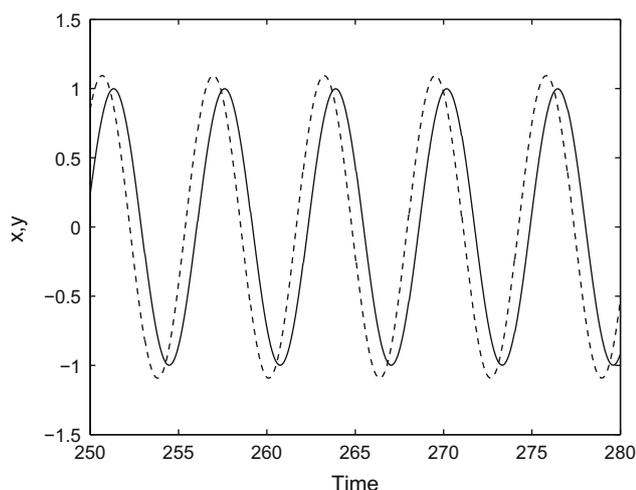


Fig. 11. Simple harmonic oscillator driving a chaotic oscillator. Harmonic oscillator parameters:  $k = 1$ , Rössler parameters:  $a = b = 0.1$ ,  $c = 14$ ,  $\tau = 0.6$ .

2-dimensional or 3-dimensional — we now have a 5-dimensional system.

It comes as no surprise that anticipating synchronization obtains with coupling in the other direction as well. Any organism, multicellular or single-cell, when taken as a dynamical system, will have a very large number of degrees of freedom. It is conceivable that some pertinent part of the environment has relatively lower dimension (e.g., illumination). This case is shown in Fig. 11.

A hypothetical biological slave system is given further latitude by the condition that only a small subspace, or even a single state variable, need participate in the proper master–slave organization. In this way, all levels of a biological system may exhibit proper organization, including organism, organ, cell, and gene. This issue is taken up below, in the discussion of anticipating circadian synchronization.

## 7. Hallmarks of strong anticipation

We can now strengthen the definition of strong anticipation given above by the enumeration of several necessary conditions for anticipation to be considered strong.<sup>4</sup> Features common to the outfielder and laser examples enrich the definition, as do extensions of anticipating synchronization to encompass delays in either master or slave system. As will become evident, it is in the character of these common features that any one implies the others. Such co-implication is true to the spirit of strong anticipation (collective, high level, non-analytic solutions) and, in some sense, ought not to be otherwise. Being definitional, it is further claimed that these conditions are jointly sufficient.

Accordingly, strong anticipation:

1. *Is an achievement by the system as a whole.* As already noted and schematized in Fig. 1, a weakly anticipatory system is minimally composed of two separate systems A (e.g., agent) and B (e.g., environment). System A, endowed with an internal model, predicts the future of system B. The property of weak anticipation is a property of system A. In contrast, the property of strong anticipation is a property of the larger system C composed from systems A and B. We can see this hallmark in the general form of delay-coupled systems expressed in Eq. (3). Any effort to separate the equations will destroy the slave's anticipatory ability. Anticipation, in this sense, comes directly from the coupling term  $k(x - y_\tau)$ .

<sup>4</sup> Many of these hallmarks will evoke features of similarly minded theories, such as those of Brooks (1991), specifically hallmarks 4 and 5, and Bickhard (2009), hallmarks 1–3. The similarity between Strong Anticipation and Brooks' "intelligence without reason" is clearly stated by p. 230 Nijhawan (2008a): "This suggests an approach that may be termed *anticipatory behavior without internal models*, which is reminiscent of a previous approach to *intelligent behavior without internal representations* Brooks (1991). "The emphasis put on process and interaction by Bickhard (2009) is also in agreement with much of the following.

2. *Is owed to proper organization.* Given the first feature, if anticipation is not a property of system A as such, but a property of C composed from systems A and B, then great importance must be placed on the way that A and B relate. In effect, the status ‘strongly anticipatory system’ is owed to proper organization. The strongly anticipatory system C must be organized internally in such a way as to support anticipation. This internal structure usually presents itself to us as a coupling between A and B. In Hallmark 1, we see that a coupling term involving both  $x$  and  $y$  is required. Here, we make this claim more specific. The coupling term must take a particular form. In the case of Eq. (3), this is  $k(x - y_\tau)$ . For example,  $k(y_\tau - x)$  will not suffice.
3. *Uses the natural unfolding of events.* Viewing the components of a strongly anticipatory system may not reveal anything special. The components are doing essentially what they should do—the ball flies through the air and the outfielder runs with eyes on the ball. The achieved anticipation inheres in the unfolding of these events. Features 1 and 2 identify that the anticipatory power of strong anticipation comes from moving the burden of anticipation from one or more individual components to the organization of the components. For the general case, this move allows individual components to operate relatively freely. It also allows individual components to be ignorant. That is, there is no requirement that individual components need to be knowledgeable of anticipation, or even of the roles they play in the larger whole. Reducing requirements for what components need to know reduces the taking out of intelligence-loans (Dennett, 1978; Keijzer, 2001; Kugler, Kelso, & Turvey, 1980; Turvey, Shaw, Reed, & Mace, 1981). In weakly anticipatory systems, loans of intelligence (knowledge of, and ability to use, the facts and principles of environment and body) must be taken by the theorist to ensure the predictive competence of the internal model. Taking these loans typically presumes resolution of major epistemological paradoxes and problems (e.g., induction problem, frame problem); (Bickhard, 2004, chap. *The Dynamic Emergence of Representation*, Fordor, 2000). It is not clear how they will be repaid. In the particular case of Eqs. (5) and (6), it is evident that Rössler dynamics are playing out as they usually do. It is the coupling term which sits between them doing all of the anticipatory heavy lifting. It is clear that this coupling term is not a model of either system, nor contains the dynamics of either system.
4. *Is purely reactive at some level of analysis.* A direct implication of retaining the natural unfolding of events is that strongly anticipatory systems are at some level purely reactive. This could be said of weak anticipation as well. That is, a model is put in place, which reacts to current conditions, and makes changes that, if the model is right, correctly respond to future states. For weak anticipation, however, the model is constructing a future. Strong anticipation does not need to bother with the

future. Simply looking at any of the systems’ details above, it is clear that the position of each state at any given time is a function of states at previous times, not future times. Thus, these systems are, at this particular level of analysis, completely reactive.

5. *Relates implicitly to future states.* The final implication of all previous hallmarks is this: strong anticipation relies on (or creates) an implicit relation to the future. This is in direct opposition to a hallmark of weak anticipation, which uses explicit references to the future. For example, a model will explicitly say that in 5 s, 5 days, or 5 years, the anticipated system will be at some state. Strong anticipation, in contrast, does not explicitly concern itself with the future. Strongly anticipatory systems go about their normal functioning, and are implicitly affected by the future because of how they are put together. The delay-coupled systems themselves, all those of the form of Eq. (3), do not contain any reference to future times, as described in Hallmark 4. A solution to such systems, however, is  $y(t) = x(t + \tau)$ , which *does* contain a reference to the future. It is in this way that future states are implicit. The nature of the coupling function in the specific cases above finds this solution automatically.

## 8. Anticipating circadian synchronization

The general supposition behind weak anticipation, that a behavior is mediated by an internal representation, is made all the more attractive when the behavior is produced or displayed in the absence of the environmental conditions in which it normally occurs—when it is manifest in a vacuum, so to speak. In a number of respects, such vacuum activity qualifies as the prime example of a “representation-hungry” problem (Clark, 1997). Perhaps its most universally profound biological instantiation is the synchronization to a cycle with a period of approximately 24 h that occurs in the absence of the cycle. As is well known, such cycles are referred to as *circadian* (derived from “circa” meaning “approximately” and “dies” meaning day) with the most prominent being the cycle of illumination—the day–night cycle. The representation-hungry nature of synchronization to the day–night cycle is that it is manifest in conditions of continuous day (that is, continuous light) or continuous night (that is, continuous dark).

Circadian synchronization is significant to the theory of cognitive systems in two ways. First, as a seemingly obligatory case of the internalization of external constraints, circadian synchronization justifies identifying internal models broadly to include characteristic phenomena of perception and cognition (Shepard, 1984). Second, as an intensively studied phenomenon (Refinetti, 2006), the detailed database on circadian synchronization introduces the varied properties that a proposed internal model mediating vacuum activity must encompass.

There is a third and, perhaps, even more important way in which the phenomenon of circadian synchronization can

illuminate inquiry into cognitive systems. Within the standard representational perspective, it can be viewed an example of one representation-hungry problem whose solution, through an internal model, provides the basis for solving multiple other representation-hungry problems, through additional internal models.

Consider the homeostatic regulation of metabolism, physiology, and behavior. As originally conceived, homeostasis is a reactive process (Cannon, 1926). Conditions inducing deviation from equilibrium (a set point) directly incur changes that counter the deviation, restoring equilibrium. Control strictly follows conditions. As currently conceived, homeostatic processes are additionally proactive, and perhaps primarily so (Moore-Ede, 1986). The effects of restorative changes are often significantly delayed requiring proactive (anticipatory) operations to ensure that the effects occur when needed, and not minutes, hours, or days, later.

Anticipatory homeostasis is seemingly grounded in one or more circadian cycles (Davidson, Castanon-Cervantes, & Stephan, 2004; Moore-Ede, 1986). In the standard perspective, to internalize an environmental cycle is to acquire a time-keeping ability. It is an ability that could be used to support a class of internal models that predict how far in advance specific homeostatic changes should be triggered to ensure that the organism is in the appropriate state at the appropriate time. The core body temperature and plasma cortisol level in humans increase in the hours before the light (or before awakening) but on different schedule—plasma cortisol increase is triggered earlier and occurs at a faster pace (Moore-Ede, 1986). The available evidence suggests potentially many different schedules of homeostatic anticipation at the level of the whole organism, at the level of the individual organs, and at the level of cells composing the organ (e.g., the liver;) (Davidson et al., 2004). The available evidence also raises the possibility that anticipatory circadian synchronization at various  $\tau$  values may characterize prokaryotic as well as eukaryotic life (Kondo et al., 1993; Steunou et al., 2006).

The foregoing identifies what can be usefully termed *anticipating circadian synchronization*. It is paradigmatic of anticipatory behavior in that the anticipation typically spans multiple subsystems at multiple length and time scales with each subsystem presenting a representation-hungry problem. Metaphorically speaking, given a clock and the charge of ensuring that  $n$  different events, with their own characteristic inertias, co-occur at, say, 2 pm, one would have to start them at different times prior to 2 pm. One could stagger them right if one had full knowledge of the dynamics of each event (that is, if one had a model of each event) and was endowed with the means to calculate (to infer) the right start-time for each event. In sum, anticipating circadian synchronization illuminates the possibility that weak anticipation entails solutions to multiple, nested, representation-hungry problems.

An inventory of the major features of anticipating circadian synchronization includes the following: (a) it is a very

general capability, spanning basic life functions and exhibited by most life forms; (b) it exhibits an array of anticipating times; (c) it can be shifted in time by experimental shifts in the 24-h cycle after some period of exposure to the new regime; (d) it persists for some duration when isolated from the 24-h cycle, typically suggesting a clock endogenous to the individual organism, organ or cell; (e) it persists in isolation as a dissociation of previously phase- and frequency-locked anticipating times. The fact of (a) is sufficient motivation for considering the possibility that the phenomenon is an instance of strong rather than weak anticipation. The analyses below suggest that features (b)–(e) are formally features of strong anticipation. They are also intended to suggest that the features may not be special to the 24-h cycle but are general to master and slave systems in “proper organization”. The analyses are conducted primarily using the coupled Rössler system of Eqs. (5) and (6).

### 8.1. Feature (b): an array of anticipating times

Using the slave-delayed general coupling of Eq. (3), it is easy to see the possibility of coupling many independent slaves to the same master. To make this clear, consider many systems coupled together in the following way:

$$\begin{aligned}\dot{x} &= f(x) \\ \dot{y}_i &= f(y_i) + k(x - y_{i,\tau})\end{aligned}\quad (10)$$

Because the coupling is unidirectional, an arbitrary number of slave systems  $y_i$  may be coupled to  $x$ . For an anticipating synchronization system such as Eq. (10), the fixed point corresponding to the anticipation time is a function of  $\tau$ . This yields the cognate comprehension that an arbitrary number of slave systems allows an arbitrary number of anticipating times. Fig. 12 provides an example within

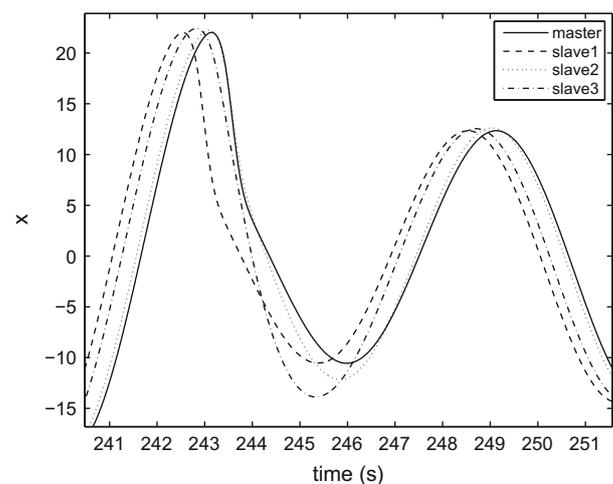


Fig. 12. By varying intrinsic dynamics of each slave system, including intrinsic delay, many slaves may show anticipating synchronization with respect to a single master system. Differing delay parameters produce a variety of phase differences. Master:  $a = b = 0.1$ ,  $c = 14$ ; Slave 1:  $a = b = 0.1$ ,  $c = 14$ ,  $\tau = 0.6$ ; Slave 2:  $a = b = 0.1$ ,  $c = 15$ ,  $\tau = 0.1$ ; Slave 3:  $a = b = 0.1$ ,  $c = 16$ ,  $\tau = 0.4$ .

the coupled Rössler system of three slaves coupled to a single master.

8.2. Feature (c): a shift of rhythm

The ubiquitous effect of a change in the period of the day–night cycle is a corresponding shift in the specific biological circadian rhythm under study (e.g., core temperature, glucagon concentration). The shift is not instantaneous, but occurs over some amount of time (Moore-Ede, 1986). In the coupled Rössler system the equivalent of a change in the external rhythm is a change in the master system. The independence of the master dynamics from the slave dynamics means that it is possible to initiate the master–slave coupling dynamics with one master system and then to observe the evolution of a new master–slave coupling dynamics following transition to a second master system.

A simulation is shown in Fig. 13. The time series begins at a point where a slaved system has undergone anticipated synchronization with a master system and is then coupled to a second master system separate from and independent of the first. Fig. 13 indicates (a) anticipated synchronization with the first master system prior to an abrupt switch to the second master system (analogous to an abrupt change of the day–night cycle) and (b) a gradual emergence of anticipated synchronization with the second master system (analogous to a new circadian rhythm).

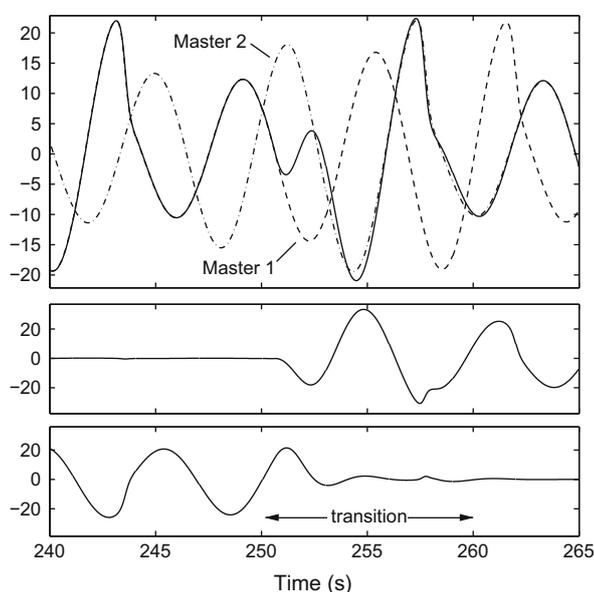


Fig. 13. (Top panel) A graph of two master Rössler systems, and one slave. By time 240, Slave has synchronized with the future values of Master 1. To improve clarity, Slave has been shifted forward in time so that during synchronization, master and slave are overlapping. At time 250, Slave is abruptly switched to couple with Master 2. (Middle Panel) The difference between Master 1 and Slave shows that before time 250 they are well synchronized, but diverge after time 250. (Bottom panel) The difference between Master 2 and Slave is graphed, showing that after time 250, there is a transition period where Slave becomes synchronized with Master 2. Simulation parameters:  $a = 0.15$ ,  $b = 0.2$ ,  $c = 10$ ,  $k = 1$ ,  $\tau = 0.6$ .

The instance of predictive homeostasis depicted in Fig. 13 continues to abide by the hallmark characteristics of strong anticipation. In particular, the slaved system is simply reacting to current conditions in some way, albeit more complexly than in the prior instances.

8.3. Feature (d): persistence after decoupling

As underscored in the introduction to this section, the key element of weak anticipation accounts of predictive homeostasis is the assumption of an endogenous circadian rhythm. Fig. 14 shows the master and slave dynamics of the coupled Rössler system when  $k > 0$  (specifically,  $k = 1$ ) and when  $k = 0$ , that is, when master and slave are coupled and when they are uncoupled, respectively. The values of the  $k$  parameter are analogous to the presence and absence of contact with an environmental dynamic (such as the day–night cycle). Inspection of Fig. 14 shows the continuation of anticipating synchronization for a definite period of time following the decoupling of master and slave.

The coupling of two 3-dimensional systems into a 6-dimensional system changes the attractor layout in which the states evolve. What was stable before the coupling might not be stable after the coupling. Likewise, what is stable when the systems are coupled may not be stable after the systems are decoupled. Persistence, in this context, is simply the transient of the slave system relaxing back toward its intrinsic stable configuration. The length of the relaxation time determines the amount of time the slave might be considered to persist.

Persistence, or relaxation time, following decoupling depends on the match between slave and master parameters ( $a, b$ , and  $c$ ). With unequal system parameters, slave and master do not share a common state space but their spaces

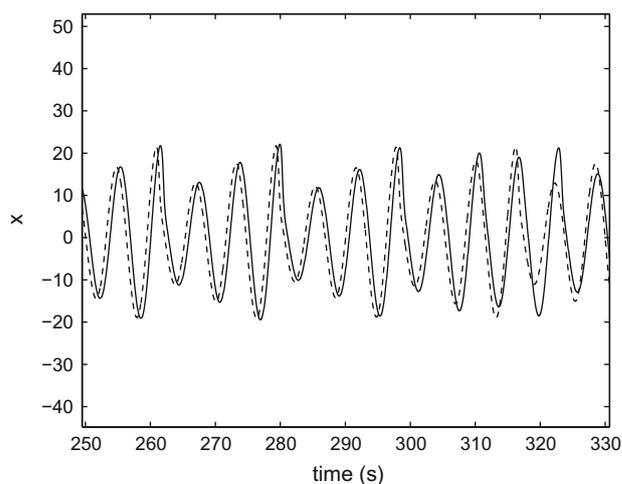


Fig. 14. At time 250, the slave system (dashed) is decoupled from the master system (solid). After a period of persistence, the slave begins to depart from the master. Parameter matching between master and slave determine the duration of persistence, that is, how long the slave's rhythm will appear “endogenous”. Master:  $a = b = 0.1$ ,  $c = 14$ ; slave:  $a = 0.1$ ,  $b = 0.13$ ,  $c = 14$ ,  $\tau = 0.6$ ,  $k = 1$ ,  $t < 250$ ,  $k = 0$ ,  $t > 250$ .

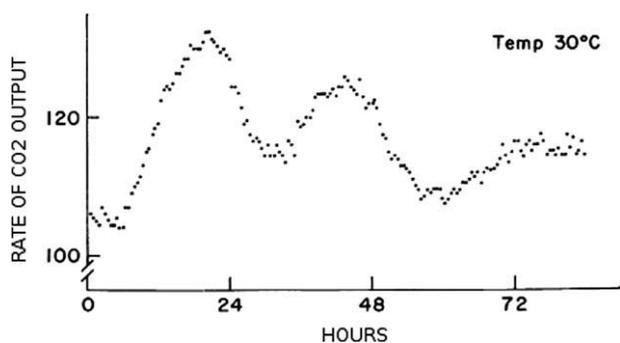


Fig. 15. CO<sub>2</sub> output of *L. perpusilla*, showing gradual dampening of circadian synchronization. (Adapted from Hillman (1970).)

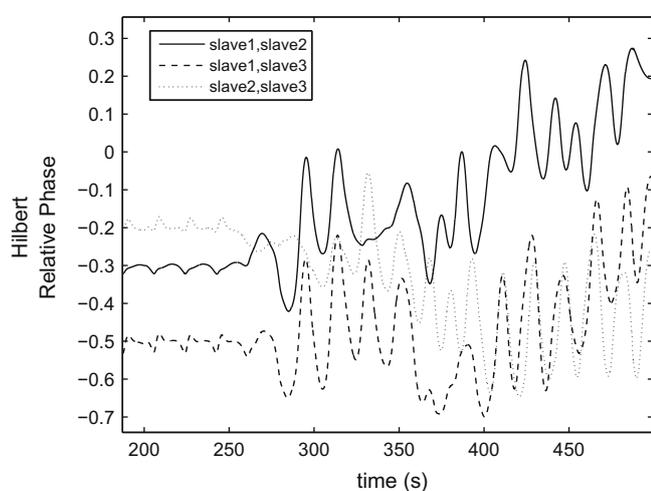


Fig. 16. While coupling to the master is in effect, phase relations between slaves are relatively constant. All slaves are decoupled at time 250. After a period of continuation, each phase relation begins to wander. Hilbert relative phase is a continuous measure of oscillator phase relations.

can be sufficiently similar to promote long relaxation times. Differences in persistence observed in experimental investigations of decoupling from externally defined rhythms (Ikeda, Sagara, & Inoue, 2000) might reflect analogous slave–master differences in the dynamics of the specific slave systems under study.

In fact, long running persistence following decoupling, while not unusual, is not the typical case. “A gradual, or even abrupt, loss of rhythmicity has been reported for various species maintained in conditions of constant darkness or constant light. p. 221 (Refinetti, 2006)”. Data collected in several instances (Pfeffer, 1875; Hillman, 1970; Hoban, Levine, Shane, & Sulzman, 1985) is quite similar in character to the behavior of our simulated systems in Figs. 13 and 14. For example, see Fig. 15 adapted from Hillman (1970).

#### 8.4. Feature (e): dissociation of previously phase- and frequency- locked anticipating times

Each slave system coupled to the master may have its own, different, intrinsic dynamics. While synchronized with the master, however, each will maintain some phase

relationship determined by relative values of  $\tau$ . After being decoupled, the slaves will relax via some transient toward their respective intrinsic behaviors as shown in Fig. 16. The common chicken provides an example (Winget, Card, & Pope, 1968). When exposed to conditions of constant (red) light, the rhythms of heart rate, locomotor activity, and deep body temperature lose their relative synchronization, with damping occurring at the slowest rate for the latter.

#### 9. Might learned homeostatic responding be an instance of strong anticipation?

To reiterate, a hallmark of strong anticipation is that nothing special, nothing extraordinary, is required of the master and slave as such. Strong anticipation is consequent to the ordinary functional modes of master and slave entering into a “proper organization”. An ordinary functional mode of biological systems is Pavlovian learning. The evidence suggests that it is a ubiquitous and versatile means of adapting an organism to its surroundings (Turkkan, 1989). The common characterization of Pavlovian learning is that it is a matter of associating a neutral or arbitrary stimulus (the eventual conditioned stimulus, CS) with an unconditioned stimulus, US. A less common characterization is that it is a matter of learning how to cope with the US (Domjan, 2005; Hollis, 1997). In this functional view, the CS helps with the coping. Consider exposure to cold. The exposure induces a drop in body temperature, the US, which is responded to by compensatory shivering and vasoconstriction, the unconditioned response, UR. If the US is prefaced with reasonable regularity by at least one CS, then the compensatory adjustments, normally reactive to lowered body temperature, can occur proactively. That is, the UR can anticipate the US (Riccio, MacArdy, & Kissinger, 1991).

The preceding example is one of many. The benefits bestowed upon an animal by Pavlovian learning are very often those of anticipation. Could Pavlovian learning be an instance of anticipatory synchronization? A model of conditioned regulation proposed by Dworkin (1993) suggests that it could. The model can be expressed by a system of equations showing the interdependence of the conditioned response,  $V$  (Eq. (11)), and unconditioned response  $S$  (Eq. (12)).<sup>5</sup>

$$V_{t,j} = V_{t,j-1} + \sum_{i=1}^m K_i S_{t+i-1,j} \quad (11)$$

$$S_{t,j} = T \left( Dstrb(t) - V_{t,j-1} - \sum_{i=1}^n a_i S_{t-\delta-i,j} \right) \quad (12)$$

Dworkin’s model is a discrete recursive system over two indices:  $t$ , the time within a trial, and  $j$ , the trial number.

<sup>5</sup> The referenced work contains a slightly different system. Eqs. (11) and (12) come from personal correspondence (2008-03-30).

Both  $t$  and  $j$  can be considered quantities of time, although they vary over differing time scales.<sup>6</sup> Here,  $Dstrb(t)$  is the *disturbance function*, which represents a disturbance from outside, or an “initiating event”;  $K$  and  $a$  are weighting functions which, roughly, take the place of salience from the Rescorla–Wagner model (Rescorla & Wagner, 1972). Finally,  $T$  is the so-called *characteristic function* of the response.

For us, the significant feature of this model is that, over time,  $V$  comes to anticipate the disturbance function  $Dstrb(t)$ . The situation appears similar in kind to the anticipatory systems examined above, that is, evolution to a negative phase relationship after a transient, incorporating some form of delayed feedback. It is expected, then, that Eq. (11) should be expressible in such a way that it shows a delay-coupled relationship to  $Dstrb(t)$ .

Using Eq. (12) to expand Eq. (11) and rearranging terms, we see how  $V_{t,j}$  depends on  $Dstrb(t)$ .

$$V_{t,j} - V_{t,j-1} = - \sum_{i=1}^m \sum_{k=1}^n K_i a_k S_{t+i-k-\delta-1,1} + \sum_{i=1}^m K_i (Dstrb(t+i-1) - V_{t+i-1,j-1}) \quad (13)$$

The second term of Eq. (13) is a delay-coupling arrangement, consisting of the difference between an external state at some time, and an internal state from further in the past. That such an arrangement is realized in a model developed to address a fundamental aspect of the adaptation of organisms to their surroundings lends a measure of support to the hypothesized generality of anticipating synchronization and the theory of strong anticipation that it represents. At a minimum, it invites pursuit of parallels between the phenomena of Pavlovian learning and those of anticipating synchronization.

## 10. On movement emulation, internal models, and strong anticipation

Having detailed anticipating synchronization as a reasonably well-developed form of strong anticipation, we can now reconsider the notion of internal model that defines weak anticipation. We do so in terms of internal model as expressed through the influential conception of motor emulation (Clark & Grush, 1999; Grush, 2004; Kawato, 1999; Wolpert & Ghahramani, 2000).

### 10.1. On full-blooded and not full-blooded internal models

It is commonly understood that time delays are intrinsic to neural processes (Nijhawan, 2008b). For the movement scientist, a particularly significant delay is that of the

proprioceptive signal, the information from mechanoreceptors about the postures and motions of limbs. The usual understanding about delayed feedback in its many manifestations is that, beyond some threshold delay, it leads to system destabilization. In the case of fast, voluntary movement the conventional wisdom is that there is an additional consequence of the intrinsic time-delay of feedback: its receipt by the controller is too late to benefit control. It is suggested therefore that to achieve a rapid movement of the requisite quality the controller must be proactive because the time scales of events (fast evolving movement versus slow feedback consequences of the movement) prohibit the controller from being reactive. An ability to simulate or emulate the proprioceptive details of the rapidly unfolding movement would help. So-called forward models are promoted for this purpose (Miall & Wolpert, 1996). As a causal representation of a limb, a forward model can reproduce the limb’s dynamics given the motor commands to the limb (generated by an inverse model) and the limb’s current state. More specifically for current purposes, it can provide a kind of mock feedback as surrogate for the actual feedback—a representation of a fast voluntary movement’s time evolution able to support quick anticipatory adjustments that ensure control.

Given that the internal forward model is hypothesized as the desideratum for proactive adjustments to slowly arriving feedback, the observed fact of a smoothly executed, rapid voluntary movement in the face of delays is prima facie evidence for the internal model’s existence. In Clark and Grush’s p. 10 Clark and Grush (1999) terms, evidence for a movement emulator is evidence for a close approximation to a “full-blooded internal representation”. The qualifier “close approximation” is warranted because the surrogate movement-with-feedback operates slightly ahead of the actual movement-with-feedback (that is, in a small negative phase relationship; see Section 2). The internal representation of the moving limb is coupled to the moving limb. For Clark and Grush (1999), to refer to an internal representation as full-blooded is to imply that the representation can be fully de-coupled from the relevant external states of affairs. The behavioral criterion is the vacuum activity presaged in the introductory remarks on anticipating circadian synchronization (Section 8).

### 10.2. Motor emulation as anticipating synchronization

It would seem that a *not full-blooded* internal representation can be characterized by a *closed loop system that exhibits anticipatory behavior* and a *full-blooded* internal representation can be characterized by an *open loop system that exhibits anticipatory behavior*. In the section on anticipating circadian synchronization (Section 8), one could identify Fig. 12 with the characterization of not full-blooded and Fig. 14 with the characterization of full-blooded. It is the case, however, that the behaviors exhibited in the two figures—anticipating circadian synchronization in the presence and in the absence, respectively, of the relevant

<sup>6</sup> The two indices may be collapsed by noting that the tuple  $(t, j)$  can be transformed into “global” time by the mapping  $(t, j) \rightarrow j_{start} + t$ . Here,  $j_{start}$  is meant as the time trial  $j$  begins. If trials have a defined length and are regularly spaced,  $j_{start}$  may, of course, be computed from  $j$  and those known quantities.

master (external) states of affairs—are outcomes of one and the same “proper organization” of master and slave systems. The lesson to be learned, perhaps, is that any proposed internal representation is, at best, a *not full-blooded* internal representation.

That said, we emphasize that anticipating synchronization, while satisfying behaviorally the above characterization of a not full-blooded representation, differs from the example that is seemingly provided by fast voluntary movement. It is a closed loop system that exhibits anticipatory behavior without an internal model. To coin a phrase, it is reactively proactive. Its application to movement amounts to replacing a process (forward modeling) compensating for intrinsic time delays by a process (anticipating synchronization) based in intrinsic time delays. The application changes the focus from predictions of the limb's dynamics *a priori* to the placing of subsystems into an organization from which prediction-like outcomes arise from the subsystems' ordinary modes of functioning *a posteriori*. There is some implicit support for the foregoing re-conceptualization in the internal model literature itself. It has, on occasion, looked to the Smith predictor (Smith, 1957; Miall, Weir, Wolpert, & Stein, 1993), a device that duly respects the positive side of intrinsic time delays.

“The Smith predictor also includes an explicit delay mechanism that delays a copy of the rapid sensory estimate to allow temporally synchronous comparison with the actual sensory consequences of the movement. . . . By ensuring synchrony between the delayed output of the forward model and the actual feedback, the Smith predictor effectively isolates the feedback delays from the control loop p. 1269 (Miall & Wolpert, 1996).”

Replacing the predicting forward model by strongly anticipatory dynamics is not a particularly large step. Both are capable of state estimation and prediction, explaining many of the same observed phenomena. Viewing the anticipatory system as a proper coupling relationship rather than as a predicting device removes the need to endow that system with detailed knowledge of its dynamics and the dynamics of the systems with which it interacts. A promissory note is that well advertised problems associated with inverse and forward models for motor control (Hollerbach, 1990) can be avoided. They include:

1. Incompleteness of the dynamic model of the plant (e.g., a limb, the body, a hand-held object).
2. Different initial conditions from those identified in the original specification of the motor commands.
3. Non-modeled perturbations that deflect the movement trajectory.
4. Complexity of inverse dynamics and mechanoreceptor activity may exceed the ability of the central nervous system to compute fast enough for purposes of control, even if sufficiently accurate internal models could be formulated.

### 10.3. An array of intrinsic delays and anticipating times, revisited

Adding to the complexity identified in Point 4 above is the fact that inverse dynamics are present not at one level, but at multiple nested levels, distinguished by their spatial and temporal scales. As pp. 738–739 Foisy and Feldman (2006) have noted:

“... the system needs to solve an exponentially increasing number of redundancy problems related to the necessity of transforming the computed torques into individual muscle forces, EMG signals, postsynaptic potentials of hundreds of individual motoneurons and millions of individual synaptic potentials descending to motoneurons from the brain. . . .”

The fact of many nested inverse-dynamics problems exacerbates Point 4. Most particularly, it brings into question the viability for biological movement systems of non full-blooded representations and weak anticipation (Ostry & Feldman, 2003; Pilon & Feldman, 2006; Turvey & Fonseca, 2008). For extra emphasis, consider the proliferation of, and challenges for, forward modeling posed by the level-dependent intrinsic time scales, from individual muscle forces to individual synaptic potentials. The current formulation of motor emulation is limited to muscle-level time delays, but for self-consistency it ought to presume emulators at all levels. Failure to do so paints a picture of weak anticipation at the muscle-level resting on strong anticipation at the other levels. To argue for emulators at all levels, however, would only magnify further the concerns raised in Point 4.

Repeating the strategy suggested above, of replacing “forward model” by “strongly anticipatory system”, but now at all levels, promises a more self-consistent and coherent account. We confronted the issue of multiple, nested intrinsic delays and the necessity of level-dependent anticipating times in Section 8. We identified many different schedules of homeostatic anticipation at the level of the whole organism, at the level of the individual organs, and at the level of cells composing the organ, and we posed the question of how those different schedules might arise. Eq. (9) and Fig. 12 revealed how an arbitrary number of slave systems can be coupled to the same master system so as to anticipate the master system at times appropriate to their individual defining dynamics. The developing theory of fast voluntary movements might benefit from this scheme.

## 11. Conclusions

(p. 177) Dennett (1991) has remarked, pursuant to Valéry (1919), Jacob (1982), that “[t]he fundamental purpose of brains is to *produce future* (italics added)”. In the present article we have considered a very small sample of the extensive variety of instances in which organisms can be said to produce future, instances to which labels such as anticipatory and predictive readily apply. We have considered our

small sample from the perspective of Dubois' (2003) notion of strong anticipation. The strategic importance of this notion is that it invites a shift of focus from theorizing about the relation between representation and anticipation to theorizing about the relation between coupling and anticipation. Rather than asking how the future is produced from an internal model, one asks about the coupling (between organism and environment) that results in coordination with the future. Paralleling Dennet's remark, but borrowing more directly from (p. 66) Jacob (1982), it might be preferable to say that: "one of the deepest, most general functions of living systems is to coordinate with the future".

The phenomenon of anticipating synchronization that arises from time delays in appropriately coupled dynamical systems provides both a spur and a means for thinking about coordinating with the future. Offered as an alternative to the internal model (Stepp & Turvey, 2008; Stephen, Stepp, Dixon, & Turvey, 2008), anticipating synchronization presents a framework within which to (a) critically evaluate the meanings of "internal", "model", and "prediction" when incorporated into analyses of living systems and their cognitive achievements and (b) advance inquiry into the more general thesis of strong anticipation. In respect to (a) and the notion of prediction, consider once again the instance of coordinating with the future shown in Fig. 10. Around time 265, the chaotic master abruptly changes its trajectory. The change "fools" the slave, which responds by producing a substantial "error". Analogues of phenomena common to prediction, namely, incorrect prediction and manifest surprise are exhibited. The implication is that anticipation by relatively low-level systems sans model-based prediction is a potential source of insight into apparent future producing by (much higher level, much more abstract) cognitive systems.

## Acknowledgement

Preparation of this manuscript was supported by NSF Grant SBR 04-23036 to M. T. Turvey and NICHD Grant HD-01994 to Haskins Laboratories. We thank Stephanie Petrusz for valuable discussions. Correspondence concerning this article should be addressed to Nigel Stepp, Center for the Ecological Study of Perception and Action, 406 Babbidge Rd., U-1020, Storrs, CT 06269-1020; email: nigel.stepp@uconn.edu.

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