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RESEARCH NOTE

Anticipation in feedback-delayed manual tracking of a chaotic oscillator

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Abstract Manual tracking of non-chaotic targets, with and without feedback delay, as well as discrete prediction of chaotic maps have each been demonstrated by humans. Feedback-delayed tracking of chaotic targets, on the other hand, has not been well investigated. To this end, 10 participants were asked to track a chaotically moving target presented on a computer display by means of controlling a similar on-screen object using a pressure sensitive tablet and hand-held stylus. The participants were given delayed visual feedback of their own movements. Task success subsequently required anticipation on the part of the participant. Using 6 values of delay from 20 ms to 1 s, evidence shows that (a) participants are able to synchronize with a chaotic target, even with some amount of applied delay, (b) task performance varies systematically with applied delay, and (c) this same systematic dependence is predicted for systems exhibiting anticipating synchronization.

Keywords Synchronization · Feedback delay · Anticipation · Manual tracking

Introduction

To synchronize one's own movements to the motions of the surrounding environment, one must appropriately control the movement of, for example, one's limbs. In the instance of manual tracking, the task demands that the motion of the hand becomes synchronized with the motion of a target. Thus, synchronization tasks and motor control are intricately linked.

Investigation into the ability of humans to synchronize with a dynamical system has, for the most part, developed along two major axes. First, the time series presented to a participant may be discrete or continuous,[1] e.g., a metronome (Repp 2005) or smooth oscillation (Vercher and Gauthier 1992). Second, the underlying system may be regularly periodic (Voss et al. 2007) or chaotic (Neuringer and Voss 1993; Smithson 1997; Heath 2002). Stochastic signals are another option at this point, but for the moment we restrict ourselves to periodic and chaotic signals (i.e., deterministic signals).

These two dimensions outline four areas of synchronization research, such as continuous-periodic (Voss et al. 2007) and discrete-chaotic (Stephen et al. 2008). In all of these cases, humans are able to synchronize in some respect with the time series presented to them (for a particularly non-human case, see Saigusa et al. 2008). That synchronization exists, however, is not an adequate description of the results of the foregoing line of inquiry—some form of anticipation is nearly always observed as well. An anticipatory "systematic error" has been well documented in at least the discrete tasks (Fraisse 1984; Radil et al. 1990; Aschersleben and Prinz 1995) and explicit investigations show anticipatory ability in both discrete and continuous ones as well (Metzger and Theisz 1994; Foulkes and Miall 2000).

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[1] Most continuous time series are in some way discrete if they are produced by numerical methods such as sampling or simulation. For present purposes, discrete signals approximating continuous ones are considered continuous.

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Delayed feedback puts participants into a position where they must anticipate in order to succeed at a task (Vercher and Gauthier 1992; Foulkes and Miall 2000; Voss et al. 2007). This anticipation-engendering property of delay is reminiscent of a phenomenon in dynamical systems known as anticipating synchronization (Voss 2000). State-based synchronization of two dynamical systems x and y may not only be complete ($y(t) \approx x(t)$) or lagged ($y(t) \approx x(t-\tau)$), but also anticipating ($y(t) \approx x(t+\tau)$). A class of unidirectionally coupled systems that often results in anticipating synchronization is shown in Eq. 1.

$$\begin{aligned} \dot{x} &= f(\mathbf{x}) \\ \dot{y} &= g(\mathbf{y}) + k(\mathbf{x} - \mathbf{y}_\tau) \end{aligned} \tag{1}$$

where $f(\mathbf{x})$ and $g(\mathbf{y})$ are the intrinsic dynamics of $\mathbf{x}$ and $\mathbf{y}$, k is a coupling strength, and $\mathbf{y}_\tau = \mathbf{y}(t-\tau)$ or in other words, the state of y delayed by τ in some unit of time. The vectors $\mathbf{x}$ and $\mathbf{y}$ themselves are states of a master and slave system, respectively.

The similarity between Eq. 1 and many synchronization tasks involving delay invites direct comparison (as in Stepp and Frank 2009; Voss et al. 2007). Therefore, this particular study has three aims. First, to show that synchronization to a continuous-chaotic oscillator is possible. Second, to show that there is a systematic dependence of synchronization behavior on applied delay. Third, to link performance of feedback-delayed synchronization tasks by humans to the phenomenon of anticipating synchronization.

Methods

Participants

Ten (five female and five male) undergraduate and graduate students at the University of Connecticut participated in this study. Of the 10, nine were right handed and one left handed. Right and left handedness was defined by the hand with which the participant preferred to draw. Participants gave informed consent and, in the case of undergraduates, received class credit for their voluntary participation. The study was approved by the University of Connecticut Institutional Review Board, and conducted in accordance with the Declaration of Helsinki.

Design

Each participant viewed a computer display (39 cm diagonal, 1,280 × 800 pixel resolution) at a distance of approximately 65 cm from screen to eye. A pressure sensitive tablet (18 cm diagonal) sat 30 cm in front of the same display. Participants held a 14 cm stylus in their dominant hand that they could position on the tablet in order to interact with the display. The tablet and stylus were visible to the participant, and the background color of the display was set to a light gray color.

Trials, each lasting 80 s, were organized into three blocks of six for a total of 18. In general, there was a 2–3 second gap between each trial, although participants were able to rest between trials whenever they wished. During each trial a 20 × 20 pixel blue square, the target, moved along a chaotic ellipsoid trajectory. The on-screen s_x and s_y coordinates of the trajectory were generated by the x_1 and x_2 dimensions of a “chaotic spring” system specified by Eq. 2.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\left(2\pi\left(\frac{x_3}{\alpha} + \beta\right)\right)^2 x_1 \\ \dot{x}_3 &= -x_4 - x_5 \\ \dot{x}_4 &= x_3 + a x_4 \\ \dot{x}_5 &= b + x_5(x_3 - c) \end{aligned} \tag{2}$$

This particular system maintains an elliptical oscillation, at the same time varying chaotically in both amplitude and frequency. Therefore, the trajectories produced are chaotically hard to predict, but remain trackable by naive participants. Dimensions x_3, x_4, and x_5 comprise a standard Rössler oscillator. This chaotic system then drives the stiffness of a simple harmonic oscillator, dimensions x_1 and x_2. For all trials, $a = b = 0.1$, $c = 14$, $\alpha = 100$, and $\beta = 0.3$. The system described by Eq. 2 is then a straightforward extension of simpler systems that might produce more common circular or linear trajectories.

Procedure

At the beginning of each trial, a 160 s time series was simulated from initial conditions $x_1 = 1$, $x_2 = 0$, $x_4 = 3.432$, $x_5 = 20.9$, and x_3 taken from a uniform distribution on the interval [18.5, 19.5]. The first 80 s of this time series was truncated in order to remove any transient behavior. Lastly, x_1 and x_2 were mapped to on-screen coordinates s_x and s_y by the mappings in Eqs. 3 and 4.

$$s_x = \frac{(s_{\text{width}} - 2s_{\text{xpad}})(x_1 - \min x_1)}{\max(x_1 - \min x_1)} + s_{\text{xpad}} \tag{3}$$

$$s_y = \frac{(s_{\text{height}} - 2s_{\text{ypad}})(x_2 - \min x_2)}{\max(x_2 - \min x_2)} + s_{\text{ypad}} \tag{4}$$

where $s_{\text{xpad}} = 0.25 s_{\text{width}}$ and $s_{\text{ypad}} = 0.45 s_{\text{height}}$. While the simulation was displayed on-screen, participants used the stylus and tablet to control a 10 × 10 pixel green square with the instruction to keep the green square in contact with the blue, ideally overlapping.[2] The movement of the stylus, however,

[2] If overlapping, the participant’s square is drawn on top of the target square.

was delayed before being displayed to the participant. Delays were randomized within each block from the set $\tau = \{20, 200, 400, 600, 800, 1{,}000\}$ ms. Horizontal and vertical coordinates of the undelayed tablet input, i.e., the movement of the hand, were captured as y_1 and y_2, respectively.

As such, the data collected parallel the states **x** and **y** of the master–slave system described in Eq. 1. Coupling, and subsequently synchronization, is considered to be between the hand and target. The delayed movements of the participant's green square play the same supporting role as does y_τ from Eq. 1.

Data analysis

For the purpose of analysis, the first dimension of the target time series, x_1, was compared to the first dimension of the participant time series, y_1. These two dimensions correspond to the horizontal movements of each. To determine both the level of synchrony, ρ, and amount of phase shift, τ^*, between x_1 and y_1 we used the maximum of the cross-correlation between the two (Stepp and Frank 2009).

For each trial, these two quantities were calculated according to Eq. 5.

$$\rho = x\text{corr}(\tau^*) = \max x\text{corr}(h) \tag{5}$$

where $x\text{corr}(h)$ is the normalized cross-correlation function of x_1 and y_1 with lags from the interval $h = [-40, 40]$.

Results

In general, participants reported experiencing only three different delays: no delay, small delay, and large delay. Participants also consistently reported trailing behind the target even when motions of the hand were clearly anticipatory. Figure 1 shows typical target and participant time series over a single 80 s trial.

Each participant produced three blocks of six time series such that each τ condition was repeated three times. While the first block was considered practice and not analyzed, the second two blocks were analyzed using the methods above to generate a ρ and τ^* measure for each trial. Participants in similar tasks (Miall and Jackson 2006) have shown adaptation across many trials. In the case of the current task, however, differences between participant performance in block 2 and 3 are negligible. As such, analyses below are conducted using mean values per participant. Given our two measures, we may examine the effect of τ on each in turn.

Dependence of synchronization on delay

As expected, participants attained higher synchrony for smaller delays than for larger delays. Maximum cross correlations between hand and target (ρ above) ranged from 0.98 (SD = 0.016) at 20 ms delay to 0.65 (SD = 0.070) at 1,000 ms delay. Performance measures for all delay conditions are plotted in Fig. 2a. All pairwise comparisons (paired-sample t-tests) between τ conditions are significant at $\alpha = 0.02$. Less trivial than decrease in performance with τ is the particular shape of the curve. Dependence of ρ on τ shows a significant cubic trend, $F(1, 9) = 5.40$, $p = 0.045$. Differences between ρ values are smallest for small and large τ values.

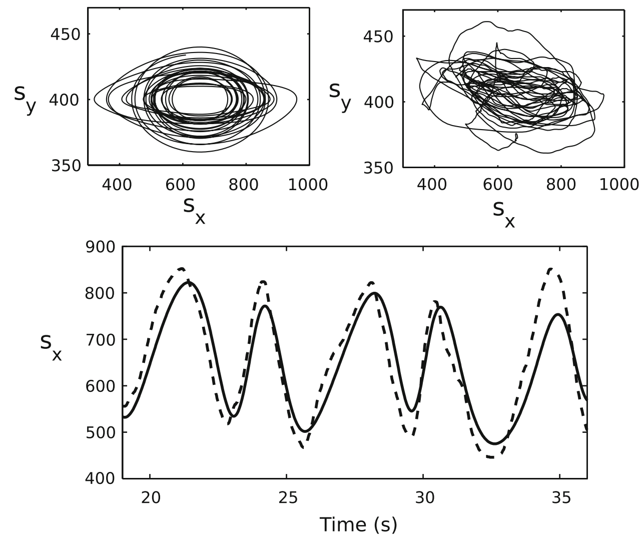

Fig. 1 Typical time series for both target (*top left*) and participant (*top right*). In this particular trial, the target time series was generated with $x_3(0) = 18.6386$, and the participant's response was subjected to a delay of $\tau = 400$ ms. The bottom panel contains a plot of the evolution of target (*solid*) and participant (*dashed*) together in time. A slight negative phase shift is evident, owing to the fact that the participant's stylus is leading the target by approximately 180 ms on average

Dependence of phase shift on delay

At 20 ms delay, participants' phase shifts are significantly negative, $t(9) = -2.48$, $p = 0.035$, meaning that their hands were trailing the motion of the target. At 200 and 400 ms, however, phase shifts are significantly positive, identifying anticipation by the hand in those conditions, $t(9) = 4.74$, $p = 0.0011$ and $t(9) = 4.88$, $p < 0.001$, respectively. Up to this point, τ^* varies roughly linearly with τ, $R^2 = 0.54$, $F(2, 27) = 32.42$, $p < 0.001$ with τ as a predictor ($b = 0.44$, $p < 0.05$). At delays of 600 ms and above, however, variability across participants becomes quite large and any systematic dependence of phase shift on τ, as well as significance of either positive or negative shifts, disappear. Phase shifts for each τ condition are plotted in Fig. 2b.

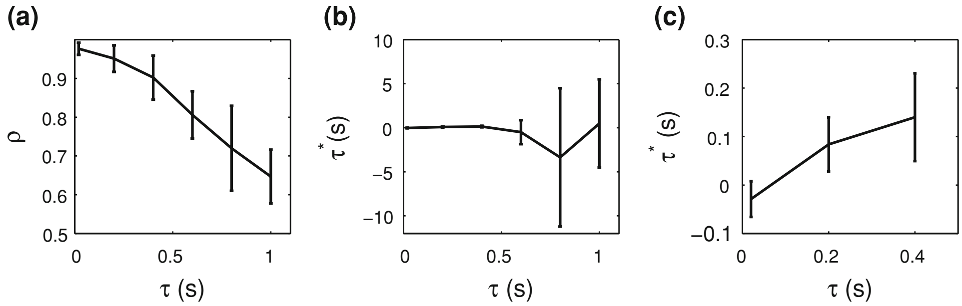

Fig. 2 **a** Dependence of maximum cross correlation, ρ, on delay, τ. The value of ρ represents the highest correlation between participant and target, allowing for an arbitrary phase shift. Note that the negative slope doubles after 400 ms. **b** Dependence of phase shift, τ^*, on delay, τ. For $\tau \geq 600$ ms, phase shifts are most likely artifacts of using Eq. 5 on time series which are not well correlated. **c** Dependence of τ^* on $\tau \leq 400$, highlighting a linear regime

Discussion

This study set out to show that synchronization between hand and continuous-chaotic oscillator—even with applied delay—is possible, to show a systematic dependence of synchronization behavior on that delay, and to link that behavior to similar behavior within the phenomenon of anticipating synchronization. The results above immediately apply to the first two aims, showing that participants can indeed track chaotic movements and that their ability to do so depends systematically on applied delay. The particular form of this dependence leads to consideration of the third aim. Before addressing this last point, however, there are several elements of the first two worth examining more closely.

Anticipation of the target

It is clear that participants were able to behave in anticipation of the target in the 200 and 400 ms τ conditions. Phase shifts in these conditions are significantly positive. As described earlier, positive phase shifts of this sort denote anticipation. In the 20 ms condition, however, participants trailed the target, a somewhat unintuitive result given the existence of anticipation in conditions with much greater delay. At 600 ms delay and beyond, participants cease to reliably lead or trail the target. This is the first hint that there may be a critical τ at some point between 400 and 600 ms.

Is there a critical value of τ?

It is not only the fact of anticipation that is of interest, but the existence of a systematic dependence of both synchronicity and phase shift on delay. Several properties distinguish themselves in Fig. 2. There are qualitative differences present in each graph between $\tau \leq 400$ ms and $\tau \geq 600$ ms. First considering the measure of synchronicity in Fig. 2a, this boundary separates regions of small and large variability, as well as regions of small and large changes between adjacent τ conditions. In other words, values of ρ at $\tau \leq 400$ are relatively high, cluster more tightly across participants, and do not differ greatly from one τ to another. On the other hand, values of ρ at $\tau \geq 600$ are decreasing rapidly and are highly variable across participants.

Similar differences exist in Fig. 2b. Values of τ^* at $\tau \leq 400$ also exhibit drastically lower variability than at $\tau \geq 600$. More interesting from the view of anticipatory systems is that the τ conditions for which participants were able to anticipate the target were also below this threshold, except for the very smallest τ condition, 20 ms.

In answer to the question posed by this section, it appears that there is a critical value, or region of values, $400 < \tau_c < 600$ ms. Properties of ρ and τ^* below this region are distinctly different from those above it. Further experiments should examine the space between 400 and 600 ms using a higher resolution, in order to find just how sharply defined this critical region is. For instance, Vercher and Gauthier (1992) performed a similar experiment, albeit with a non-chaotic target, using 10 values of τ from 0 to 450 ms. In that study no critical value of τ was discovered, which fits well with the observations above. It is expected that if τ values greater than 450 ms were used, a change in behavior would be observed similar to the one described here.

It is also expected that the particular critical τ suggested here is a function of target (Eq. 2) as well as participant dynamics. That is, for any instantiation of a master and slave system coupled in the way presented above, there should be a corresponding τ_c.

Properties of τ^* above and below τ_c

Below the supposed critical time delay τ_c, values of τ^* show a linear relationship with τ. This region is plotted alone in Fig. 2c. A linear relationship such as this is particularly interesting when one considers the task. As τ increases, so does each participant's τ^*. It is not the case, however, that participants anticipate by exactly τ or greater (i.e., the slope the line in Fig. 2c is less than one). The combination of these two facts amounts to the interesting result: participants demonstrate the ability to anticipate more than they actually do at a particular value of τ. Being specific, a participant tracking in the 200 ms condition will anticipate the target by some time τ_1, even having demonstrated in the 400 ms condition that he or she is capable of anticipating by $\tau_2 > \tau_1$. Participants anticipate by an amount relative to τ, not by the greatest of their ability. Values of τ^* above τ_c become two orders of magnitude more variable (mean $SD = 0.058$ for $\tau < \tau_c$, mean $SD = 2.2$ for $\tau > \tau_c$). That is, participants cease to perform reliably at these larger τ values.

Similarity to simulated anticipating synchronization

The similarity between this task and the delay coupling arrangement of Eq. 1 was outlined at the start. The task put to participants is to minimize the difference between the current position of the target and the past position of the hand-held stylus. Participants must change their behavior in the present in order to synchronize two states separated by time. To successfully do so requires anticipation of the target.

Anticipating synchronization, following Eq. 1, has stability that is dependent on the compatibility of f and g as well as the two parameters k and τ (Pyragas and Pyragiene 2008; Stepp and Turvey 2009; Toral et al. 2003; Voss 2000). Simulations which span combinations of k and τ show very similar relationships between ρ and τ as those seen in Fig. 2a, as well as linear relationships between τ^* and τ (Voss 2000). For two systems that are not perfectly matched (i.e., $f \neq g$), the profile of Fig. 2a is just what is expected for nearly constant k and varying τ (see Stepp and Turvey 2009).

The existence of a critical time delay, τ_c, is also fully expected. In simulated anticipating synchronization, such a delay tends to separate high ρ from low, positive τ^* from negative, and stability from instability (Stepp and Turvey 2008, 2009; Voss 2000). That is, just the sort of behavior seen in the analyses above. Thus, there is at least an empirical link between manual tracking with delayed feedback and the phenomenon of anticipating synchronization. For the present task, this may not be surprising provided that the task itself so closely mirrors the dynamics of Eq. 1. Particularly striking, however, is the fact that participants exhibit a τ^* according to a linear relationship with τ, rather than some higher phase shift that they are clearly able to attain. This is notable, as a participant might have deviated from behavior predicted by anticipating synchronization, but did not.

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References

Aschersleben G, Prinz W (1995) Synchronizing actions with events: the role of sensory information. Percept Psychophys 57(3):305–317

Foulkes A, Miall R (2000) Adaptation to visual feedback delays in a human manual tracking task. Exp Brain Res 131(1):101–110

Fraisse P (1984) Perception and estimation of time. Annu Rev Psychol 35:1–37

Heath R (2002) Can people predict chaotic sequences? Nonlinear Dynam Psychol Life Sci 6(1):37–54

Metzger M, Theisz M (1994) Forecast: Program to obtain forecasts from subjects for successive values of chaotic time series. Behav Res Methods Instrum Comput 26:387

Miall R, Jackson J (2006) Adaptation to visual feedback delays in manual tracking: evidence against the Smith predictor model of human visually guided action. Exp Brain Res 172(1):77–84

Neuringer A, Voss C (1993) Approximating chaotic behavior. Psychol Sci 4(2):113–119

Pyragas K, Pyragiene T (2008) Coupling design for a long-term anticipating synchronization of chaos. Phys Rev E Stat Nonlin Soft Matter Phys 78(4):046217

Radil T, Mates J, Ilmberger J, Pöppel E (1990) Stimulus anticipation in following rhythmic acoustical patterns by tapping. Cell Mol Life Sci 46(7):762–763

Repp B (2005) Sensorimotor synchronization: a review of the tapping literature. Psychon Bull Rev 12(6):969

Saigusa T, Tero A, Nakagaki T, Kuramoto Y (2008) Amoebae anticipate periodic events. Phys Rev Lett 100(1):018101

Smithson M (1997) Judgment under chaos. Organ Behav Hum Decis Process 69(1):58–66

Stephen DG, Stepp N, Dixon JA, Turvey MT (2008) Strong anticipation: sensitivity to long-range correlations in synchronization behavior. Physica A 387(21):5271–5278

Stepp N, Frank TD (2009) A data-analysis method for decomposing synchronization variability of anticipatory systems into stochastic and deterministic components. Eur Biophys J 67(2):251–257

Stepp N, Turvey MT (2008) Anticipating synchronization as an alternative to the internal model. Behav Brain Sci 31(02):216–217

Stepp N, Turvey MT (2009) On strong anticipation. Cognit Syst Res doi:10.1016/j.cogsys.2009.03.003

Toral R, Masoller C, Mirasso CR, Ciszak M, Calvo O (2003) Characterization of the anticipated synchronization regime in the coupled fitzhugh-nagumo model for neurons. Physica A 125:192–198

Vercher J, Gauthier G (1992) Oculo-manual coordination control: ocular and manual tracking of visual targets with delayed visual feedback of the hand motion. Exp Brain Res 90(3):599–609

Voss HU, McCandliss B, Ghajar J, Suh M (2007) A quantitative synchronization model for smooth pursuit target tracking. Biol Cybern 96(3):309–322

Voss HU (2000) Anticipating chaotic synchronization. Phys Rev E Stat Nonlin Soft Matter Phys 61(5):5115–5119