# Emergence of Anticipation at Multiple Time Scales

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Members of all five kingdoms (Monera, Protista, Fungi, Plantae, and Animalia) exhibit some level of anticipatory behavior. Such generality of phenomenology suggests generality of principle. Beginning from the ubiquitous concept of synchronization, the phenomenon of anticipating synchronization is taken as candidate for such a principle. Anticipating synchronization is placed within the larger context of anticipatory systems that do not employ an internal model for explicit prediction of future states (weak anticipation), but rely on lawful, reactive behavior that places a system in implicit relation to the future of another (strong anticipation). The beneficial effect of delayed feedback in this regard is discussed and a new, general form for anticipatory coupling is developed. This general form allows for the specification of certain classes of anticipatory systems with differing phenomenology. Two experiments instantiate two of these coupling classes and establish the phenomenology of anticipatory behavior in human subjects. As an initial step towards discovering a more general anticipation principle, the parallels between theoretical and empirical behaviors are discussed. Finally, the implications for anticipation as an element of organism-environment dynamics lead to Liquid State Machine reservoirs as a possible model system and tool for the investigation of important properties of anticipatory systems.

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# 1 Introduction

## 1.1 Historical Concerns

Gibson (1966b, 1966a, 1979/1986) frequently expressed concern with the uses of, and interpretations of, terms such as "expectation" and "anticipation". James (1890/2007), did likewise nearly a century before, complaining that "psychological time" is not confined to an infinitesimal present. Together, James and Gibson present a unified front against a view of both space and time as discrete, elementary containers. Instead, both advocated an idea of an organism's awareness of its context within nested, dynamic surroundings.

If perception extends into the past and future, as Gibson and James observed, then by virtue of what is the future accessible? The ecological study of perception and action makes a serious commitment to perception-action without mediation (e.g., Turvey, Shaw, Reed, & Mace, 1981). At the same time, perception-action is not isolated to the knife's edge of the present. Thus, the ecological study of perception and action must pay mind to future and past; perhaps by disposing of or redefining those very concepts.

### 1.1.1 The fiction of the specious present

James (1890/2007, p. 606) sums up the primary issue of the present rather succinctly, "The knowledge of some other part of the stream, past or future, near or remote, is always mixed in with our knowledge of the present thing." James makes the point that the present has duration; he goes on to say that the instantaneous present is "an altogether ideal abstraction". In his words, "the practically cognized present is no knife-edge, but a saddle-back, with a certain breadth of its own on which we sit perched, and from which we look in two directions into time." That is, "the present" is a vantage point in a more than analogous comparison with spatial perception. James (1890/2007, p. 610) also warms to the idea of painting perception of time on the same canvas as space, "When we come to study the perception of Space, we shall find it quite analogous to time in this regard. Date in time corresponds to position in space. . . ."

The benefit of such a perspective is that with the right principles of perception, time and space become equally manageable. The governing intuition of the present work is that Gibson (1966b, 1979/1986) describes those right principles. While he differs from James in his willingness to treat time and space, or sequence and pattern, as the same, the principles he lays out for perception of surface layouts should apply equally to a temporal dimension.

## 1.2 Strong Anticipation

Again, the problem is that ecological psychology requires a theory of anticipation that does not resort to a mediating model. In fact, it requires this for its theory of perception to be consistent. An attractive possibility is a theory that derives its power not from internal models, but from appropriate coupling and lawfulness. Dubois (2001) distinguishes between prediction of the future given a model, labeled weak anticipation and prediction of the future without a model—and relying instead on systemic lawfulness—labeled strong anticipation. If it exists, so-called strong anticipation already resembles an Ecological account of anticipation.

As defined by Dubois (2001) and as elaborated by Stepp and Turvey (2010), strong anticipation is a relatively abstract concept. So far, a useful entry point into this domain has been the more concrete anticipating synchronization (H. U. Voss, 2000). Briefly, anticipating synchronization is a phenomenon of nonlinear dynamical coupling resulting in anticipation—not from explicit prediction, but from the coupling itself. As such, anticipating synchronization is considered to be an instance of strong anticipation (Stepp, Chemero, & Turvey, 2011; Stepp & Turvey, 2008, 2010). One class of dynamical systems that robustly exhibits anticipating synchronization is

$$\dot{\mathbf{x}} = f(\mathbf{x})$$
$$\dot{\mathbf{y}} = g(\mathbf{y}) + k\left(\mathbf{x} - \mathbf{y}_\tau\right) \tag{1.1}$$

where $f$ and $g$ are intrinsic dynamics of systems $\mathbf{x}$ and $\mathbf{y}$ respectively, $k$ is coupling strength, and $\mathbf{y}_\tau$ represents delayed feedback. Delayed feedback of the type in Eq. (1.1) results in a minimization between the states $\mathbf{x}$ and $\mathbf{y}_\tau$. That is, minimization between the current state of $\mathbf{x}$ and a previous state of $y$. If the difference between these two states is successfully minimized, then it must be the case that the difference between the current state of $\mathbf{y}$ and a future state of $\mathbf{x}$ is also minimized. In this way, $\mathbf{y}$ comes to be synchronized with the future of $\mathbf{x}$.

An example of anticipating synchronization is shown in Fig. 1.1. Here, $f$ corresponds to the Rössler oscillator and $g$ to a simple linear spring, as in the system below.

$$\dot{x}_1 = -x_2 - x_3$$
$$\dot{x}_2 = x_1 + ax_2 \tag{1.2}$$
$$\dot{x}_3 = b + x_3(x_1 - c)$$

$$\dot{y}_1 = y_2 + k\left(x_1 - y_{1,\tau}\right)$$
$$\dot{y}_2 = -wy_1 \tag{1.3}$$

### 1.2.1 Strong Anticipation in Humans

In order to test the consistency of this approach in explaining human behavior, Stepp (2009) investigated anticipation in a manual tracking task. In this experiment, a target moved on a computer display in a chaotic elliptical pattern specified by the $x_1$ and $x_2$ states of Eq. (1.4). Parameters $\alpha$ and $\beta$ allow the

Figure 1.1: A Rössler attractor ($x_1$, solid) driving a linear spring ($y_1$, dashed) according to Eqs. (1.2) and (1.3). The anticipating synchronization arrangement enables the driven system, after a transient of a few cycles, to anticipate the driver.

tuning of frequency; $a$, $b$, and $c$ are parameters typical of the Rössler attractor comprising states $x_3$, $x_4$, and $x_5$. Participants were tasked with tracking this target by controlling a cursor with a hand-held stylus. Varying delay between movement of the stylus and movement of cursor also varies the degree to which participants must anticipate the motion of the target.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\left(2\pi\left(\frac{x_3}{\alpha} + \beta\right)\right)^2 x_1 \\
\dot{x}_3 &= -x_4 - x_5 \\
\dot{x}_4 &= x_3 + a x_4 \\
\dot{x}_5 &= b + x_5(x_3 - c)
\end{aligned}
\tag{1.4}
$$

Stepp (2009) found that human behavior in this task is consistent with behavior predicted by Eq. (1.1). Two kinds of experiments, similar in design to that of Stepp (2009), would provide useful extensions of the preceding finding. Each is previewed below.

**An experiment on continuous tracking (Experiment 1).** In a tracking experiment, participants view a computer display showing a moving target. In order to reduce the space of possible explanation, the trajectory of the target is specified by the $x_1$ state of Eq. (1.4). In this way, participants are not able to anticipate by, for instance, matching frequency then maintaining a phase difference. Additionally, since Eq. (1.4) is not periodic, a phase lag is properly distinguishable from a phase advance. The display also includes an object that the participant controls by moving a stylus across a pressure sensitive tablet; this object is the participant's cursor.

Figure 1.2: Temporal relationships in a navigation paradigm. The black keyhole shape represents the participant's position. The point at which the middle of the road scrolls past this cursor at time $t$ is $x(t)$. The position of the participant's hand is assumed to be some intended position subject to reaction time delay $\tau_r$, $y(t - \tau_r)$. If the position of the cursor is subject to additional delay $\tau_d$, the location of the cursor is $y(t - \tau_r - \tau_d)$. A point on the path corresponds to a future value of $x$, namely $x(t + \tau_m)$.

The participant's task is to intercept the target with a stream of upward moving dots emitted from the cursor. Speed of these dots determines the amount that a participant must anticipate in order to succeed at the task. Time series data of target and cursor motion can then be subjected to cross-correlation analysis and model comparison.

**An experiment on navigation (Experiment 2).** An experiment of the first kind is a test of what kinds of models can account for simple tracking behavior in humans. A richer experience for the participant allows the experimenter to connect with many more facets of ecological theory. For example, a winding path can be calculated according to the $x_1$ state of Eq. (1.4). A slice of this time series can be presented to the participant as a roadway using a perspective transform. The speed of a virtual vehicle along the path can be fixed, and direction of travel can be controlled using a stylus and tablet. Horizontal position of the stylus on the tablet can be mapped to a steering angle in the interval $\theta$ radians. The charge for the participant is to keep the vehicle "on the road".

This rudimentary driving simulator adds an additional degree of freedom for the participant. Since the path has extent, altering one's gaze to look further down the path allows one to look further, in a sense, into the future. The temporal relationships between various components of this paradigm are shown in Fig. 1.2.

An eye tracker can used to capture the direction of the participant's gaze, transformed into on-screen coordinates. The vertical gaze position, with appropriate

Figure 1.3: Two ways to have long and short paths within a network of interacting nodes.

transformation, can be made to correspond to $\tau_m$ as described in Fig. 1.2. Results from an experiment of this kind, which are focused on a particular treatment of anticipation, can be put into context with other treatments within the steering literature (Kim & Turvey, 1999; Land & Horwood, 1995; Wilkie, Wann, & Allison, 2008).

## 1.2.2 Multiple Time Scales

In the theory and experiments introduced above, the power of a single feedback delay is evident. This single delay allows anticipation on a single time-scale. That is, for delay $\tau$ anticipation up to $\tau$ is possible. Further, the theory and experiments introduced above focus on a single master system, perhaps with its own characteristic time scale. In a more realistic vein, thinking of a real perceiving-acting organism or agent, there are likely to be an indefinite number of possible "master" systems to choose from–each with some set of characteristic time scales.

From a network perspective, one may conceptualize delayed feedback as a single signal propagating along two paths, a slow path and a fast path. One way to accomplish this, if the signals propagate at the same speed, is for one path to be longer than the other. It is also possible for one path to contain more segments than the other. In this latter case it is possible to consider networks where propagation delay within a segment is negligible but propagation from segment to segment takes some amount of time. For a schematic description, see Fig. 1.3.

For these systems, delay time is equivalent to difference in segment number. That is, if the fast path is 3 segments, the slow path might be 8 segments, resulting in a 5-segment delay.

In order to move from simple dynamical systems to embedded, complex systems and communities, the lessons learned from theory and experiment should be applied at the system level. A convenient structure for speaking of organism and environment relations at the system level is the so-called CES model (Mahner & Bunge, 1997). This model seeks to minimally describe a system in terms of components (C), environment (E), and structure (S). The CES model, however, is a static description. To extend the model to a dynamical system, Frank (2010) takes a tack similar to Beer (1995) and writes CES in a dynamical way. This

dynamical CES, or DCES, maintains a system level description, but allows for this description to change over time.

Judging by some low-level communities of organisms that show anticipatory behavior (such as amoeba, Saigusa, Tero, Nakagaki, & Kuramoto, 2008), another attractive model system is randomly connected recurrent networks, of which liquid state machines are a common example. A liquid state machine (Maass, Natschläger, & Markram, 2002; Burgsteiner, Kröll, Leopold, & Steinbauer, 2007), or LSM, is a randomly connected graph, where nodes receive time varying input from their incoming connections and produce time varying outputs on their outgoing connections. While similar in principle to a neural network, LSMs are much more abstract and are not necessarily meant to be analogous to a biological network of any sort. Being randomly connected, LSMs are recurrent. Additionally, connection gains or weights are fixed upon creation of the network. The structure of LSMs results in a highly non-linear spatio-temporal pattern among the nodes with even a single input. The main graph portion of an LSM is known as its reservoir, activation spreading through the graph in analogy to ripples spreading on the surface of a liquid. Due to these properties, the reservoirs of an LSM are prime candidates for investigation into the emergence of anticipation.

The LSM concept appears to fit naturally into the CES framework. The potential benefit of such a fit follows from the expectation that the anticipatory dynamics described above are encapsulated in an LSM reservoir. Accordingly, articulating LSMs in the CES model could provide a way to ground those phenomena to the minimal foundation that the model represents.

# 2 Generalized Anticipatory Coupling Function

## 2.1 Formal Description

In one of the original descriptions by H. U. Voss (2000), anticipating synchronization results from two dynamical systems coupled via a particular coupling function. Generally we may consider coupling to come from some function $h(x, y, t)$, where in Eq. (1.1),

$$h(x, y, t) = k(x(t) - y(t - \tau)) \tag{2.1}$$

In Eq. (2.1), there exists a single delayed feedback to $y$, which is coupled to a single point in time (the current time) of $x$. As suggested in Chapter 1, however, a richer feedback and coupling structure is possible by providing more complicated or general coupling functions. In the case of delayed feedback, we may recognize that may take on an arbitrarily long value, or even multiple values. Formally, having multiple instances of delayed feedback is represented as a sum of many couplings. Each coupling has the same form as Eq. (2.1). For a set of discrete delays, there are as many terms in the sum as delays. Generally, however, the space of possible delays is continuous, and Eq. (2.1) can be extended to

$$h(x, y, t) = \int_0^\infty K(s) \left( x(t) - y(t - s) \right) \, ds \tag{2.2}$$

where $K(s)$ is now a coupling strength function, rather than a simple gain factor, and $s$ takes on values of delay in $y$. Such an arrangement is formally similar to a distribution of random delays (Cushing, 1977), which have been shown to damp chaotic behavior (Thiel, Schwegler, & Eurich, 2003) as well as synchronize coupled systems at lower than normal coupling strengths (Sen, Dodla, & Johnston, 2005). Atay and Karabacak (2006, p. 523) note, "To the extent that multiple delays in maps can be considered as the counterpart of distributed delays, one might anticipate further stabilization effects in such general networks."

The second kind of experiment identified in Chapter 1, allows us to consider cases where coupling may exist to upcoming states of $x$, as is the case for coupling to a path laid out in space. Considering these future values of $x$, we may again generalize in likewise fashion to obtain

$$h(x, y, t) = \int_0^\infty \int_0^\infty K(s, u) \left( x(t + u) - y(t - s) \right) \, ds \, du \tag{2.3}$$

where $u$ now takes on possible future times of $x$. Note that the limits for both integrals are zero, so that $t + u$ and $t - s$ remain disjoint. While it is possible to

have chosen a coupling function in which the time arguments of $x$ and $y$ overlap, a positive time difference between master and slave times is fundamental for the existence of anticipating synchronization (Stepp & Turvey, 2010). As such, we keep the time domains of $x$ and $y$ separate. Given this generalized coupling function, we may explore a space of possible arrangements by exploring the space of coupling strength functions $K(s, u)$. For instance, we may recover the coupling function in Eq. (1.1) by choosing

$$K(s, u) = k\delta(s - \tau)\delta(u) \tag{2.4}$$

where $\delta$ is the Dirac delta function. In a similar fashion, we may choose a continuous range of delays,

$$K(s, u) = k\left(H(s) - H(s - \tau)\right)\delta(u) \tag{2.5}$$

where $H(x)$ is the Heaviside step function. For clarity, note that Eq. (2.5) combined with Eq. (2.3) results in the following coupling function

$$h(x, y, t) = \int_0^\tau k\left(x(t) - y(t - s)\right)\, ds$$

Lastly, again considering an experiment of the second kind presented in Chapter 1, where road width under a perspective transform falls off like $\frac{2}{\pi}\arctan\frac{1}{d}$ after distance $d$, a possible $K(s, u)$ is

$$K(s, u) = \frac{2}{\pi}\arctan\left(\frac{1}{u}\right)\delta(s - \tau)$$

providing for the coupling function

$$h(x, y, t) = \int_0^\infty \frac{2}{\pi}\arctan\left(\frac{1}{u}\right)(x(t + u) - y(t - \tau))\, du \tag{2.6}$$

That is, a system in which there is a quickly but infinitely diminishing coupling to upcoming values along with self-feedback for a single delay. While the integral of this choice for $K(s, u)$ diverges, practical and physical limitations would constrain it in practice. More importantly, the point is that there is great flexibility in the choice of coupling arrangement. The examples above are schematized in Fig. 2.1.

## 2.2 Simulation Studies

As stated above, choosing different functions for $K(s, u)$ results in different types of delayed feedback coupling functions in support of anticipating synchronization. It is possible to have a single delay in the feedback term, several delays, or even a continuous range of delays. Likewise, it is possible to couple to the current state of the master system or to one upcoming state, many upcoming states, or a

Figure 2.1: Illustrations of three different coupling arrangements. (top) Coupling to the present state of the master with a single delayed feedback term. (middle) Coupling to the present state of the master with a continuous distribution of delayed feedback. (bottom) Coupling to a continuous section of the master with a single delayed feedback term.

range of upcoming states. In order to make discussion of these possibilities easier, a simple notation for a general class of coupling function can be defined. As $u$ and $s$ represent master and slave time-shifts, respectively, we use $U$ and $S$ to denote their place in the coupling function. A subscript denotes the multiplicity of time-shift, e.g. 0, 1, $n$, or $\infty$ for, respectively, current time, one shift, many shifts, or a continuous range. Using this notation, we may refer to the canonical coupling function from Eq. (1.1) as $U_0 S_1$, denoting coupling to current time with one delayed feedback. With this notation in hand, we now turn to simulations of combinations of interest, namely $U_0 S_n$ and $U_\infty S_1$.

## 2.2.1 Simulations of $U_0 S_n$

Simulations of the canonical delay-coupling system in Eq. (1.1) have been conducted for the Rossler-Spring system (Stepp & Turvey, 2010), and certain features of the resulting dynamics noted. Furthermore, Stepp (2009) saw evidence of these same features in empirical data for another system from the $U_0 S_1$ class. Below, we conduct simulations of a Rssler-Spring system specified by Eqs. (1.2) and (1.3) with a coupling function from the $U_o S_n$ class for values of $n \geq 1$, specifically Eq. (2.3) with $K$ given by Eq. (2.7). The Rssler system uses parameters $a = b = 0.1, c = 14$, and the Spring system uses $w = 1$. Initial conditions of both systems were kept constant between simulations, at $x_1 = 18.68, x_2 = 3.432, x_3 =$

(a)                               (b)

Figure 2.2: Measurements synchrony ($\rho$, left) and anticipation ($\tau^*$, right) of simulations of coupling class $U_0 S_n$, which has coupling to the present state of the master, and some number of discrete feedback delays. Maximum delay ranged from 0 to 2 s, and number of equally distributed delays ranged from 1 to 15. In this simulation each delay was given equal weight.

$20.9, y_1 = 1, y_2 = 0$. Simulations were run in MATLAB using the *dde23* delay differential equation solver with differing lags for each as described below.

Two dimensions along which the discrete delay set can vary are number of delays and maximum delay. For instance, the delay sets $\{0.3, 0.6\}$ and $\{0.2, 0.4, 0.6\}$ differ in number, but not maximum, while $\{0.3, 0.6\}$ and $\{0.4, 0.8\}$ differ in maximum, but not number. To cover a region in this space of possible delay sets, many simulations were run with maximum delay and feedback count combinations taken from $\tau = \{0.1, 0.2, \ldots, 2\}$ and $n = \{1, 2, \ldots, 15\}$. For each combination, a delay set was constructed by choosing $n + 1$ equally spaced delays from 0 to $\tau$, then dropping 0. For this collection of simulations, each delayed feedback term was given equal weight scaled by the number of feedback terms. In terms of Eq. (2.3), the system being simulated is given by

$$K(s, u) = \sum_i^n \frac{1}{n} \delta(s - \tau_i) \delta(u) \tag{2.7}$$

with $\tau_i$ taken from the $n$-element delay set as constructed above.

Fig. 2.2a shows the maximum cross correlation measurement (Stepp & Frank, 2009) for each combination of feedback count and maximum delay. This measurement is the highest correlation found at some time shift between two time series. The time shift at which that happens is plotted in Fig. 2.2b. Please note that the canonical case is present in here for $n = 1$. One clear feature of the plot is that correlation remains high for higher delays when adding more feedback elements. That is, adding more delays stabilizes synchronization. Again, this is consistent with other reports in the literature.

(a)                    (b)

Figure 2.3: Measurements synchrony ($\rho$, left) and anticipation ($\tau^*$, right) of simulations of coupling class $U_\infty S_1$, which has coupling to a continuous section of the master, and a single discrete feedback delay. Feedback delay ranged from 0 to 2 s, and maximum look-ahead time ranged from 1 to 8 s.

## 2.2.2 Simulations of $U_\infty S_1$

In the following simulations, a single delayed feedback is combined with coupling to a continuous section of upcoming values of the master system. This arrangement is not supported explicitly by the *dde23* solver used in previous simulations, which only handles positive delay values. In order to conduct the simulation, a master time series was solved using an ordinary differential equation solver and the solution used within the slave DDE equations to later evaluate future values as needed.

Again, there is a range of possible values for both feedback delay and amount of look-ahead. Rather than taking all upcoming values into account, a section of the master time series is considered. In equation form, the system being studied here is given by

$$K(s, u) = \frac{2}{\pi} \arctan\left(\frac{1}{u}\right) (H(u) - H(u - \tau_m)) \, \delta(s - \tau_d) \qquad (2.8)$$

or, plugging in to Eq. (2.3), as an evaluated coupling function,

$$h(x, y, t) = \int_0^{\tau_m} \frac{2}{\pi} \arctan\left(\frac{1}{u}\right) (x(t + u) - y(t - \tau_d)) \, du \qquad (2.9)$$

where $\tau_m$ is taken from the set $\{1, 2, \ldots, 8\}$ and $\tau_d$ is taken from $\{0.2, 0.4, \ldots, 1\}$. The $\rho$ and $\tau_*$ measures are presented in Fig. 2.3.

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# 3 Experiment 1

## 3.1 Introduction

Chapter 3 is an implementation of the first of the two forms of experiments previewed in Chapter 1. It is clear from previous studies of human manual tracking that tracking can be anticipatory. Delayed feedback puts participants into a position where they must anticipate in order to succeed at a task (Vercher & Gauthier, 1992; Foulkes & Miall, 2000; H. Voss, McCandliss, Ghajar, & Suh, 2007). As noted in Chapter 1, there is some evidence (Stepp, 2009) to suggest that the relationship between delay and anticipation is similar to a phenomenon in dynamical systems known as anticipating synchronization (H. U. Voss, 2000). State-based synchronization of two dynamical systems x and y might not only be complete $(y(t) \approx x(t))$ or lagged $(y(t) \approx x(t - \tau))$, but also anticipating $(Y(t) \approx x(t + \tau))$. Anticipating synchronization, in one form, is instantiated by Eq. (1.1) in which the vectors $x$ and $y$ are states of a master and slave system, respectively.

Stepp (2009) showed the dependence of anticipatory tracking on applied delay. In Eq. (1.1), however, feedback delay $\tau$ is one of two important coupling parameters, the other being coupling strength $k$. As the paradigm used in Stepp (2009) does not easily admit variable coupling strength, a new paradigm was selected. In the new paradigm, feedback is discretized, and coupling strength is assumed to scale with frequency of feedback. The ability to vary $k$ allows one to explore a $(k, \tau)$ parameter space.

Discrete feedback additionally allows for dealing with multiple delays. A single delay is covered by the coupling function above, $k(x - y_\tau)$. Allowing for multiple feedback states, each with some delay, the coupling function can be generalized along the lines identified in Chapter 2 into class $U_0 S_n$ using a discrete sum.

$$K(s, u) = \sum_{i}^{n} k_i \delta(s - \tau_i) \delta(u) \qquad (3.1)$$

for some number of delays represented by $\tau_i$, and coupling weights $k_i$.

## 3.2 Method

### 3.2.1 Participants

Sixteen students at the University of Connecticut participated in this study. The participants were 7 women and 9 men of which 12 where undergraduate students and 4 were graduate students. Of the 16, 15 were right handed and one was

left handed, identified by the hand with which the participant preferred to draw. Participants gave informed consent and, in the case of undergraduates, received class credit for their voluntary participation. The study was approved by the University of Connecticut Institutional Review Board.

### 3.2.2 Design

Each participant viewed a computer display (39 cm diagonal, $800 \times 600$ pixel resolution) at a distance of approximately 65 cm from screen to eye. A pressure sensitive tablet (18 cm diagonal) sat 30 cm in front of the same display. Participants held a 14 cm stylus in their dominant hand that they could position on the tablet in order to interact with the display. The tablet and stylus were visible to the participant, and the background color of the display was set to a light gray color given by RGB triplet (200, 200, 200).

Trials, each lasting 80 s, were organized into 3 blocks of 8 for a total of 24. Typically, there was a 4 s gap between each trial, although participants were able to rest between trials whenever they wished. During each trial a $20 \times 20$ pixel blue square, the target, moved along the top of the screen according to a "chaotic spring" function. Specifically, the on-screen $s_x$ coordinate of the trajectory was generated by the $x_1$ dimension of the system specified by Eq. (3.2).

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\left(2\pi\left(\frac{x_3}{\alpha} + \beta\right)\right)^2 x_1 \\
\dot{x}_3 &= -x_4 - x_5 \\
\dot{x}_4 &= x_3 + \alpha x_4 \\
\dot{x}_5 &= b + x_5(x_3 - c)
\end{aligned}
\tag{3.2}
$$

This particular system maintains a relatively periodic oscillation, at the same time varying chaotically in both amplitude and frequency. Therefore, the trajectories produced are hard to predict in the chaotic sense, but remain trackable by naive participants. Dimensions $x_3$, $x_4$, and $x_5$ compose a standard Rössler oscillator. This chaotic system then drives the stiffness of a simple harmonic oscillator, dimensions $x_1$ and $x_2$. For all trials, $a = b = 0.1$, $c = 14$, $\alpha = 100$, and $\beta = 0.3$. The system described by Eq. (3.2) is then a straightforward extension of simpler systems that might produce more common sinusoidal or linear trajectories.

At the beginning of each trial, a 160 s time series was simulated from initial conditions $x_1 = 1$, $x_2 = 0$, $x_4 = 3.432$, $x_5 = 20.9$, and $x_3$ taken from a uniform distribution on the interval $[18.5, 19.5]$. The first 80 s of this time series was truncated in order to remove any transient behavior. Lastly, $x_1$ was mapped to on-screen coordinates $s_x$ by the mappings in Eq. (3.3).

$$
s_x = \frac{(s_{width} - 2s_{xpad})(x_1 - \min x_1)}{\max(x_1 - \min x_1)} + s_{xpad}
\tag{3.3}
$$

where $s_{xpad} = 0.25 s_{width}$ and $s_{width}$ is screen width.

While the simulation was displayed on-screen, a stream of dots was constantly emitted from a $10 \times 10$ pixel green square, the cursor, at a variable speed and frequency. Participants were instructed to use the stylus and tablet to control the cursor to intercept the target with as many dots as possible. Each time a dot intercepted the target, it briefly changed color to red, and a score was incremented by 5 and displayed immediately above. The time taken for a dot to travel from the cursor to target defines a delay, and therefore an amount of anticipation required to succeed at the task.

Delays were randomized within each block from the set $\tau = \{0.1, 0.2, \ldots, 0.8\}$ s, in order to cover the critical region discussed in Stepp (2009). Horizontal and vertical coordinates of tablet input, i.e. the movement of the hand, were captured as $y_1$ and $y_2$, respectively. As such, the data collected parallel the states $x$ and $y$ of the master-slave system described in Eq. (1.1). Coupling, and subsequently synchronization, is considered to be between the hand and target. The time delay between cursor and target plays the same supporting role as does $y_\tau$ from Eq. (1.1). More precisely, when there are multiple dots on screen, each $i$th dot represents a delay $\tau_i$, where $0 < \tau_i < \tau$.

This is an interesting departure from the coupling function assumed in Stepp (2009). As suggested above, this departure moves the coupling arrangement from the canonical anticipating synchronization class $U_0 S_1$ to $U_0 S_n$, which has an implication for expected results as judged by simulations in Chapter 2.

### 3.2.3 Analysis

For the purpose of analysis, the first dimension of the target time series, $x_1$, was compared to the first dimension of the participant time series, $y_1$. These two dimensions correspond to the horizontal movements of each. To determine both the level of synchrony, $\rho$, and amount of phase shift, $\tau^*$, between $x_1$ and $y_1$ we used the maximum of the cross-correlation between the two (Stepp & Frank, 2009).

For each trial, these two quantities were calculated according to Eq. (3.4).

$$\rho = \text{xcorr}_{x,y}(\tau^*) = \max \text{xcorr}_{x,y}(\tau) \qquad (3.4)$$

where $\text{xcorr}_{x,y}(\tau)$ is the normalized cross correlation function of $x_1$ and $y_1$ with lags from the interval $\tau = [-40, 40]$.

A second way to describe anticipatory performance is to not measure observed $\tau^*$ at all, but compare $x$ and $y_\tau$ directly. Using $\tau^*$ as our lag of interest, we may attain a high $\rho$ if the participant is synchronizing well at some delay (namely $\tau^*$). This is not directly related to succeeding at the task, however. A correlation, $\rho_\tau$, between $x$ and $y|\tau$ gives a direct measurement of this.

Each participant produced three blocks of eight time series such that each $(\tau, k)$ condition was repeated three times. While the first block was considered practice and not analyzed, the second two blocks were analyzed using the methods above to generate $\rho$, $\rho_\tau$, and $\tau^*$ measures for each trial. Participants in similar tasks

Figure 3.1: Cross correlation analysis of participant data in Experiment 1. Plots show synchrony measures $\rho$, $\rho_\tau$, and anticipation $\tau^*$ from left to right.

(Miall & Jackson, 2006) have shown adaptation across many trials. In the case of the current task, however, differences between participant performance in Block 2 and Block 3 are negligible. As such, analyses below are conducted using mean values per participant. Given our measures, we may examine the effect of $\tau$ on each in turn.

## 3.3  Results

Fig. 3.1 depicts $\rho$, $\rho_\tau$, and $\tau^*$ measures as described above. The dependence of these measures on $\tau$ are strikingly contrary to the expected behavior seen in Stepp (2009) and predicted by simulations in the class $U_0S_1$ (for simulations see Stepp & Turvey, 2010). This deviation, however, appears consistent with predictions from simulations of the more general class $U_0S_n$, which matches the coupling arrangement used in this experiment.

A linear regression of $\tau^*$ on $\tau < 500$ ms shows a linear fit ($R^2 = 0.7602$, $F(1,50) = 158.5$, $\rho < 0.001$) with slope 0.49 (CI: [0.4157, 0.5735]). A linear regression of $\rho$ on $\tau$ shows a moderate linear fit ($R^2 = 0.5432$, $F(1,102) = 121.3$, $\rho < 0.001$) with slope -0.2051 (CI: [-0.2420,-0.1682]), but not a cubic fit ($b_3$ CI: [-0.8505, 1.1324]).

## 3.4  Discussion

In Stepp (2009), comparisons were made between standard features of simulated anticipating synchronization and the synchronization behavior or participants in a manual tracking experiment. In the present experiment, however, the structure of delayed feedback is different due to the possibility of multiple feedback delays. Employing the notation from Chapter 2, the present experiment comes from the more general class $U_0S_n$, and its empirical results show a relatively linear dependence of $\rho$ on $\tau$, seemingly without a critical region where synchrony breaks down. Increased stability for larger values of $\tau$ is one of the predictions of Chapter 2 simulations. It is feasible that extending the range of delays would result in finding that critical region, which is a hypothesis that warrants further study.

The results from this experiment serve an important role connecting the generalized theory of anticipating synchronization to human anticipatory behavior. While experiments such as Stepp (2009) establish consistence with anticipating synchronization, those results in combination with the results of the present Experiment 1 establish a much firmer predictive landscape. Employing other modeling techniques, especially those requiring an internal model of the anticipated system, are much less likely to replicate this pattern. Instead, we see that both behaviors seen in Stepp (2009) and in the present experiment are predicted by some process of synchronization relatively similar to those described in Chapter 2, most importantly, following the same alterations to coupling arrangement.

# 4 Experiment 2: Navigation

## 4.1 Introduction

While manual tracking experiments such as Stepp (2009) and Experiment 1 of Chapter 3 are clearly anticipatory and closely match prior anticipating synchronization arrangements, they are somewhat artificial. This artificiality helps connect empirical results to formal expectations, but more natural settings should be investigated as well.

Navigating over a path at some speed is such a natural setting. Given that it is a problem faced by all animals, it can be considered fundamental. Not only is this task fundamental, it is also anticipatory, entailing traveling at speed with delayed action. In order to probe this task in humans, a driving study was conducted focusing on the interplay between control delay and anticipation.

## 4.2 Method

### 4.2.1 Participants

Eight students at the University of Connecticut participated in this study. The participants were three women and five men, either undergraduate or graduate students. All eight were right handed as defined by the hand with which the participant preferred to draw. Participants gave informed consent and, in the case of undergraduates, received class credit for their voluntary participation. The study was approved by the University of Connecticut Institutional Review Board.

### 4.2.2 Apparatus

A computer display (39 cm diagonal, $1024 \times 768$ pixel resolution) was positioned at a distance of approximately 26.7 cm (SD = 8.8 cm) from screen to eye. A pressure sensitive tablet (18 cm diagonal) sat 25 cm in front of the same display, although participants were free to move it to remain comfortable. Participants held a 14 cm stylus in their dominant hand that they could position on the tablet in order to interact with the display. The tablet and stylus were visible to the participant.

Participants viewed a rudimentary driving simulator created using the VisionEgg, PyGame, and PIL Python modules. A typical view in the simulator is depicted in Fig. 4.1. Using the hand-held stylus, participants could control the visible steering wheel. Horizontal position of the stylus on the tablet was mapped

Figure 4.1: Screenshot from the driving simulator, showing the road laid out ahead of the driver, and a small steering wheel to indicate current turning rate.

to a steering angle, $\theta$, on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ radians. This angle was used to rotate the steering wheel during the simulation and also to set the turning rate (rad/s) of an invisible virtual vehicle traveling with a constant speed, $v$. Within the simulator, a configurable delay, $\tau$, could be added between steering angle and its effect on heading direction.

Eight winding roads were created from the $x_1$ state of Eq. 3.2. To generate a road, the system was simulated for 120 s with an $x_3$ initial state chosen from a uniform random distribution $U(18.0, 19.0)$. The last 60 s of the $x_1$ time series was then used as the road path. To create a road with a left and right side, this path was copied and shifted by 0.5 and then both curves were normalized to lie within the interval $[0, 100]$.

To construct an actual simulation, a road time-series was read from a text file and a $2000 \times 100$ pixel image of the road was created. The simulator screen was partitioned into sky and ground on the top half and the bottom half of the screen, respectively. Onto this partitioning the road image was projected using a perspective transform so that it vanished at the horizon. It is worth noting at this point that viewing the upcoming road is tantamount to access to upcoming states of the road. That is, with a single delayed feedback between driver and virtual vehicle, these upcoming, essentially future, states of the road match anticipating synchronization class $U_\infty S_1$.

Eye Tracking. Before using the simulator, each participant was situated for eye-tracking using an SR Research EyeLink II eye tracking system. Preparation consisted of a standard EyeLink calibration routine started from within the driving simulator. Between each trial, calibration was checked and the participant re-calibrated if the check failed.

Table 4.1: Primary and secondary time-series for road, hand, and eye

| Time-Series | Coordinates | Description | Derivation |
|---|---|---|---|
| $H(t)$ | display - $(x, y)$ | Stylus (hand) position | Participant |
| $\Theta(t)$ | ground - $(\theta)$ | Vehicle turning rate | $(\pi/2)(H_x(t) - m)/m$ |
| $E(t)$ | display - $(x, y)$ | Gaze position | Participant |
| $D(t)$ | ground - $(\phi)$ | Vehicle heading | $\int \Theta(t)dt$ |
| $V(t)$ | ground - $(x', z)$ | Vehicle position | $\int vD(t)dt$ |
| $R(t)$ | ground - $(x')$ | Road shape | Experimenter |
| $\dot{R}(t)$ | ground - $(x'/z)$ | Road heading | $dR/dz$ |
| $\ddot{R}(t)$ | ground - $(x'/z^2)$ | Road turning rate | $d\dot{R}/dz$ |

## 4.2.3 Procedure

Before completing any experimental trials, participants were given three practice trials, each lasting 60 s. During these trials, no external delay was applied to the steering mechanism. Once familiar with the simulator in general, 16 trials were presented in a randomized order. For each trial, a delay was inserted between the movement of the on-screen steering wheel and the effect that steering angle had on the virtual vehicle heading. Similar to Experiment 1, delays were taken from the set $\tau = \{0.05, 0.1, \ldots, 0.8\}$. Once the participant signaled readiness, the simulator was started and the participant attempted to steer so as to remain in between the two lines of the virtual road (see Fig. 4.1). After 60 s, the simulator stopped and the participant was allowed to rest for as long as desired. Upon initiation of the following trial, eye-tracker calibration was checked and the next simulator run began.

## 4.2.4 Analyses

In this experiment there are three primary functions of interest, corresponding to eye $(E)$, hand $(H)$, and road $(R)$. We may think of corresponding time-series, $E(t)$ and $H(t)$, developing over time within a trial, and $R(z)$ over spatial variable $z$. In addition to these primary time-series, there are several secondary time-series of interest (see Table 4.1), virtual vehicle position $(V)$, virtual vehicle turning rate $(\Theta)$, virtual vehicle heading direction $(D)$, and derivatives of $R(z)$. In cases where a time-series has multiple components, a subscript may be used to identify a specific one, for instance $H_x(t)$ denotes the $x$ coordinate of $H(t)$. During the course of a trial, the participant is, in essence, asked to coordinate these three functions in a particular way. As such, we wish to examine the coordination, or synchronization, between each pair of time-series. Each time-series has certain characteristics described below.

The road time-series, $R(z)$, is produced by the first dimension of the now standard chaotic-spring system described by Eq. (3.2). A positive and negative bias is added to $R(z)$, which is then normalized to lie within $[0, 100]$ in order to create an

Figure 4.2: Road time-series $R(z)$ generated from Eq. (3.2). See Fig. 4.3 for definition of $x'$ and $z$.



Figure 4.3: Perspective mapping between simulation display and virtual road. Display coordinates x and y are transformed to ground coordinates x and z using horizontal and vertical gaze angles, and respectively. Because the ground coordinate system is also two dimensional, the reverse transform is possible.

enclosed road-like strip as in Fig. 4.2. Coordinates in this world-space are $(x', z)$. Spatial derivatives over $z$, $\dot{R}(z)$ and $\ddot{R}(z)$, take on the meanings of road heading and turning rate respectively.

When presented to the participant, $R(z)$ undergoes a transform composed of a rotation and translation according to the participants virtual heading and position on $R(z)$ and a perspective transform mapping $R(z)$ to a display-space with $(x, y)$ corrdinates on a viewing plane. These mappings are schematized in Fig. 4.3. The eye time-series, $E(t)$, exists in the display-space. Likewise, we can use the reverse perspective transform in order to achieve an $E(z)$. Raw $E(t)$, however, consists of gaze position on the $(x, y)$ viewing plane over time.

Finally, the hand time-series, $H(t)$, consists of the location over time of a hand-held stylus on a pressure-sensitive tablet. This 2D coordinate system is set to be the same as the viewing plane, but is mapped to an angle, $\Theta(t)$, by the following mapping,

$$\Theta(t) = \frac{\pi}{2} \frac{H_x(t) - m}{m} \tag{4.1}$$

where $m$ is half of the viewing plane width. In order to replicate the act of steering, $\Theta(t)$ is taken to be a rotation rate in radians per second.

During the course of the simulation, $\Theta(t)$ is integrated to produce a virtual

heading, $D(t)$. A velocity vector is composed of this heading and a constant speed in world units, $v$, and further integrated to produce a virtual position, $V(t)$. The position time-series $V(t)$ is now a time-series of $(x\prime, z)$ coordinates. We may also consider the time-series constructed of $V_{x\prime}(t)$ at the points $V_z(t)$, which is then directly comparable to $R(z)$.

The combination and coordination of $E(t)$, $H(t)$, and $R(z)$ allows for a situation in which multiple slave systems are being driven by one master. Such arrangements have been studied in general and with specific application to circadian synchronization by Stepp and Turvey (2010) and Stepp et al. (2011). It is also conceivable, however, that a nested arrangement exists, for instance that $R(z)$ serves as master for $E(t)$, which in turn serves as master for $H(t)$. Lastly, there may even be a combination of these arrangements such as a $R(z)E(t)$ hybrid serving as master for $H(t)$.

## 4.3 Results

### 4.3.1 Hand-Road System

Of the time-series described above, $R(z)$ is the only one that can be considered an independent variable. In the language of coupled time-series it also acts as a master system. As described above, the instructed task for the participant is to stay between the lines of the road as best as possible. That is, maintain synchrony between $V_x(t)$ and $R(V_z(t))$. Synchrony plots, the familiar $\rho$ and $\tau^*$ measurements, for this pair are shown in Fig. 4.4. Participants have, however, only one way to control $V(t)$, which is by controlling the turning rate $\Theta(t)$. Therefore, the control problem for the participant is to maintain synchrony between $\Theta(t)$ and $\ddot{R}(V_x(t))$, that is to match the turning rate of their vehicle to the turning rate of the road. Synchrony plots for this pair are shown in Fig. 4.5.

Fig. 4.5 allows for comparison with standard features of anticipating synchronization (H. U. Voss, 2000; Stepp, 2009). The measures $\rho$ and $\tau^*$ show distinctive features in their relation to $\tau$. Specifically, anticipating synchronization dynamics predict a cubic shape for $\rho$, a low-variability linear relationship of $\tau^*$ to small values of $\tau$, and sudden high-variability and weak relationship of $\tau^*$ to values of $\tau$ past some critical region. In the experiments of Stepp (2009), this critical region appeared to be between 0.4 and 0.6 s. Each of these features exists in Fig. 4.5. A linear regression of $\rho$ on powers of $\tau$ up to degree 3 shows a cubic shape ($R2 = 0.96031$, $F(1, 126) = 96.7831$, $p < 0.001$, $b_3$ CI: [0.5196, 3.805]). Values of $\tau^*$ show a linear relationship the values of $\tau$ below 0.5 s ($\tau \leq 300$ ms: $R^2 = 0.9629$, $F(1, 38) = 103.8268$, $p < 0.001$; $\tau \leq 500$ ms: $R^2 = 0.71872$, $F(1, 78) = 20.4415$, $p = 0.0019467$), at which point there is a sudden jump in variability (see Fig. 4.5c).

Fig. 4.4 on the other hand, is a metric of performance of the task goal, but is not directly related to anticipating synchronization. As such it shows some of the features of anticipating synchronization, but only weakly so. A linear regression

Figure 4.4: Cross correlation analysis comparing $V_x(t)$ and $R(V_z(t))$. This pair encapsulates the task goal presented to the participant. a) Maximum cross correlation, b) time shift at maximum cross correlation, c) standard deviation of time shifts across participants, d) initial range of time shift. Time shifts here are represented in ground units.

Figure 4.5: Cross correlation analysis comparing $\Theta(t)$ and $\ddot{R}(V_z(t))$. This pair closely matches the control problem presented to the participant. a) Maximum cross correlation, b) time shift at maximum cross correlation, c) standard deviation of time shifts across participants, d) initial range of time shift. Time shifts here are represented in ground units.

Figure 4.6: Gaze and steering angle data for a driver in a real driving setting. Reproduced from Land and Lee (1994)



Figure 4.7: Gaze location and steering angle data from a simulated driving session.

of $\rho$ up to a cubic term fits the data well ($R2 = 0.95199$, $F(1, 14) = 79.3112$, $p < 0.001$), but the cubic term does not significantly contribute to the overall model ($b_3$ CI: [-0.8146,3.524]). Values of $\tau^*$ at or below $\tau$ of 300 ms show a linear relationship ($R^2 = 0.93138$, $F(1, 4) = 54.2922$, $p = 0.0018078$), but those at or below 500 ms do not ($R^2 = 0.0057061$, $F(1, 8) = 0.045911$, $p = 0.8357$).

### 4.3.2 Eye-Road System

The behavior of the eyes while driving has been studied previously, for instance by Land and Lee (1994). There, two drivers in real-world conditions drive down a winding road while their gaze locations and steering angles were tracked. In terms of Table 4.1, $R(z)$ is defined by a real roadway, and data collected corresponded to $E(z)$ and $D(t)$. Data from Driver 1 of this study is reproduced in Fig. 4.6.

Comparing Fig. 4.7, data from the simulator used in this study, to Fig. 4.6, it appears that drivers in this simulated environment behaved similarly to Land and Lees drivers. That is, gazes fall on the upcoming turns, and steering angle follows the road. Land and Lee (1994) showed that drivers look at the road tangent point. This is also true within the simulated environment, although it appears differently in Fig. 4.7 than in Fig. 4.6. The difference is due to the portion of road visible in each case.

Approaching the Eye-Road system as before, we may conduct a usual cross correlation analysis. Comparing the road and gaze time series using a cross correlation is non-trivial since gazes do not extend monotonically along the road, but move back and forth in the z direction. One way to compare eye and road is to

Figure 4.8: Cross correlation analysis of $R(z)$ and $E(z)$ at $z$ values given by $Vz(t)$. a) Maximum cross correlation, b) gaze shift at maximum cross correlation.

take the vehicle position at each point along the road and ask at what $x'$ coordinate is the gaze location. That is, construct a time series using the $z$ coordinate of $V(t)$ and the $x'$ coordinate of $E(z)$. This time series is then compared to $R(z)$. Plots of $\rho$ and $\tau^*$ for this comparison are shown in Fig. 4.8.

Coupling between these two time series is not the type of anticipating synchronization described in Chapter 2. Nevertheless, the cross correlation analysis does show temporal relationships and synchrony. Unless the driver is looking straight down, gaze location $E(z)$ is always ahead (greater $z$ coordinate) of $V(t)$. This shows up in Fig. 4.8 as a bias toward anticipatory $\tau^*$. A notable departure from usual patterns in the $\tau^*$ plot is a near constant shift until synchronicity breaks down.

## 4.4  Discussion

The driving simulator developed for this study provides for a minimal driving environment and means to manipulate control delay. Regardless of its simplicity, the simulator generates a rich data set as summarized in Table 4.1. Together, these time series allow for detailed analysis of the eye-hand-road system in the presence of control delay. While comparison between any of the time series in Table 4.1 is possible, there are a few that are more interesting than others, especially when investigating the relationship of empirical data to theoretical expectations.

As described above, in the present experiment the participant has a single degree of freedom to control, the turning rate of the virtual vehicle. To stay on the the road, the control problem is to match this turning rate to the turning rate of the road at the current vehicle position. It is this comparison that shows the greatest correspondence to properties of anticipating synchronization, as detailed above. It is the case, however, that the coupling structure of the present experiment is not the usual single master time, single delay type. Instead, as noted

earlier, coupling is made to upcoming values as well. In Chapter 2, simulations were conducted which match this type of coupling, for different delay times and different amounts of upcoming values. These simulations help to weave together the phenomenology seen in the time-series comparisons in Fig. 4.5 and Fig. 4.8, that being the $\Theta(t)$ and $\ddot{R}(V_z(t))$ pair and the $E(z)$ and $R(z)$ pair. The shape of Fig 4.5a is expected from typical anticipation synchronization behavior, but the $\tau^*$ plot in Fig 4.5d shows much increased anticipation (anticipation by more than the imposed delay $\tau$). A second unexpected feature is that the phase shift between eye and road time-series is constant for changing $\tau$. Both of these properties are expected when taking into account the effect of coupling to upcoming values. As seen in the simulated $\tau^*$ plot of Fig. 2.3b, $\tau^*$ increases with increasing $\tau$ at a slope greater than one. Additionally, there is a look-ahead value at which anticipation is maximized across values of $\tau$. This last feature suggests that a constant shift between eye and road is also expected, if gaze direction functions to maximize anticipation. Clearly these statements deserve further study to move past the point of conjecture, but the theoretical predictions of Chapter 2 make them plausible enough to do so.

# 5 Anticipation and Organism-Environment Dynamics

Anticipation can be described as a particular arrangement between organism and environment. In simple models and simulations, such an arrangement is typically taken to be a negative relative phase or similar measurement of one dynamic behaving in accordance with the future of another. A particular conception of the relationship between organism and environment, then, has implications for a corresponding theory of anticipation. A view that places a great distance between organism and environment would require a theory of anticipation in which the organism would have a great deal of work to do in order to anticipate behavior of the environment. On the other hand, a view that treats organism and environment as more connected would have a more integrated approach to anticipation.

Organism-environment systems and their dynamics have been approached from a variety of perspectives. Some of these perspectives have relevance to general ecological concerns about how organism-environment relations should be formulated (Turvey, 2009). For instance, lack of *a priori* boundaries or granularities is one such concern. A minimal model of a system that does not take inside and outside, such as organism and environment, to be necessarily separate is the so-called CES model (Mahner & Bunge, 1997).

## 5.1 Minimal model of a system

For a system $s$, a minimal model of that system, according to Mahner and Bunge (1997), comprises components of the system $C(s)$, the environment of the system $E(s)$, and the structure of the system $S(s)$. Structure can additionally be split into internal structure $S_I(s)$, i.e. relationships between components of the system, and external structure $S_E(s)$, i.e. relationships between components of the system and the environment. The model is typically visualized as in Fig. 5.1.

The CES model, as it is known, is interesting in this context by the way it defines environment for a system. Components $C(s)$ are left somewhat nebulous, but $E(s)$ is clearly defined as just those components of the environment that interact with $C(s)$ via structural relationships $S_E(s)$. While there is no explicit dynamics in the plain CES formulation, this does impart a type of implicit dynamic to $E(s)$, if one considers that $S_E(s)$ is something that can change over time. As such, the environment and system are defined in a relative, context-dependent way. Defining an explicit dynamics for a CES-type formulation is the subject of what follows.

Figure 5.1: The CES model. Showing the relationship between components of the system $C(s)$ and the environment of the system $E(s)$. Arrows between parts of $C(s)$ and between $C(s)$ and $E(s)$ are the structure of the system $S(s)$, with thin arrows denoting internal structure, and think arrows denoting external structure.

## 5.2 Dynamics of a minimal system

Beer (1995) describes an organism environment dynamics according the diagram in Fig. 5.2. Organism and environment are modeled as two coupled nonautonomous dynamical systems. Coupling comes in two pieces, a perception function $P$ and an action function $A$. The functions $P$ and $A$ are taken broadly to encapsulate all effects that an environment has on the organism and all effects that an organism has on its environment, respectively. They are specifically not limited to the effects usually attributed to a sensor or motor apparatus.

Taking the dynamical view of Beer (1995) along with the system description of Mahner and Bunge (1997), Frank (2010) expresses the CES model in terms of a dynamical system.

$$
\begin{aligned}
d_I x_O(t) &= S^{int}(x_O)dt \\
d_E x_O(t) &= S^{ext}_{E \to O} \\
dx_O &= d_I x_O + d_E x_O \\
dx_E &= S^{ext}_{O \to E}(x_O, x_E)dt
\end{aligned}
\tag{5.1}
$$

Here $x_O$ is a state vector corresponding to $C(s)$ and $x_E$ is a state vector corresponding to $E(s)$, leaving structural relationships as the only way for states to change. This very large class of dynamical systems provides a way to deal with dynamics at the system level without making a naive separation between system and environment.

Figure 5.2: Embedded model of organism ($O$) and environment ($E$) interaction
(Beer, 1995). $O$ affects $E$ via an action ($A$) dynamic, $E$ affects $O$ via
a perception ($P$) dynamic.

## 5.3 Lessons from theory and experiment

A feature that remains constant in all of the preceding theoretical and empirical
discussions is the presence of a time shift between two dynamical systems. This
shift is often manifested by delayed feedback within a single system, but, especially
given the generalizations in Chapter 2, we see that time shifts can take many
forms.

Delay in physical systems is ever present due to the simple fact that all events
take some amount of time to progress; nothing happens instantly. With regard
to anticipation, this is fundamental. Because events take time to unfold, the
possible future of a system is constrained, and anticipation is made possible. As
such, there is a deep connection between anticipation and delay.

Additionally, we see from above that the distribution of delay matters for the
anticipatory behavior of the system. For instance, having a variety of feedback
at different delay times tends to stabilize dynamics. This is evident from the
deviation in Experiment 1 from more standard behavior (e.g. Stepp, 2009).

Lastly, it appears to be a general feature that as delay time increases, so does
anticipation time. In the original H. U. Voss (2000), this correspondence is shown
analytically, and is supported in several subsequent treatments and is evident in
Experiments 1 and 2 as well as simulations in Chapter 2.

## 5.4 Examples of many delays

In the previous section delays are observed to be important, if not fundamental,
for anticipation. Also observed is the physical systems cannot help but contain
delays. Delay is often a consequence of that fact that it takes time for matter to
move from one location to another, such delay known as transport delay. This sort

Figure 5.3: Five-node random graph. Solid lines are arcs of positive gain, dashed are arcs of negative gain.

of delay can be relatively easily described in a chemical system where transport proceeds primarily via diffusion. In such a system delay is a consequence of diffusion rate.

The slime mold plasmodium presents an example chemical system in which transport delay plays a role. During the aggregation phase of the slime mold life cycle, extracellular cAMP (cyclic adenosine monophosphate) is expelled by individual cells and adds to a collective chemotactic environment which affects all cells in the population (Gross, Peacey, & Trevan, 1976; Konijn, 1972). Diffusion of cAMP from a particular cell out into the intercellular environment is a clear example of transport delay.

The dynamics of the slime mold system, at an abstract level, consist of discrete nodes connected by a medium with transport delay. A candidate model system matching this arrangement is the reservoir of a liquid state machine. A liquid state machine (Maass et al., 2002; Burgsteiner et al., 2007), or LSM, is a randomly connected graph, the reservoir, where nodes receive time varying input from their incoming connections and produce time varying outputs on their outgoing connections. Typically, a linear output layer is trained to extract some arbitrary function from the dynamics of the reservoir. While similar in principle to a neural network, LSMs are not necessarily meant to be analogous to a biological network of any sort. Being randomly connected, LSMs are recurrent. Additionally, connection gains or weights are fixed upon creation of the network. The structure of LSMs results in a highly non-linear spatio-temporal pattern among the nodes given even a single input.

As a very small example, a 5-node random graph is shown in Fig. 5.3. Solid lines have positive gain, dashed lines have negative gain. $N1$ was defined to be an

Figure 5.4: Continuous relative phase $\phi$ between input and $N2$, showing that $N2$ has a negative phase relationship to the input node, i.e. $N2$ is leading the input.

input node, and a simulation of Eq. (3.2) was used to set its value. Values of other nodes were set according to Eq. (5.2). The continuous input from Eq. (3.1) was simulated using a discrete time integrator (Runge-Kutta) at 100 Hz. The network was updated on a sample by sample basis using this numerical simulation.

$$v_i(m) = 0.9v_i(m-1) + \frac{1}{\sum_{j=1}^{N}|G_{ji}|} \sum_{j=1}^{N} v_j(m-1)G_{ji} \qquad (5.2)$$

where $v_i(m)$ is the value of node $v_i$ at time $m$, $N$ is the total number of nodes, and $G$ is a graph matrix with entries $G_{ij} = 1$ for a positive gain connection from $i$ to $j$ and $G_{ij} = -1$ for negative gain connections.

As depicted in Fig. 5.4, $N2$ comes to anticipate the input. Fig. 5.3. in combination with the discussion of delay above provides an intuition as to why this may be. We see a short path directly connecting $N1$ and $N2$; all other connections provide various longer paths. Given the loop between $N4$ and $N5$, there is no longest path from $N1$ to $N2$.

Anticipation within the network (as some population of nodes with negative relative phase) is emergent in the "predictive, epistemological" sense of (O'Connor & Wong, 2009). The prediction from theoretical and empirical results is that, provided some distribution of feedback delays, some population of nodes will become anticipatory.

## 5.5 LSM in Dynamical CES

We may quickly identify elements of an LSM with elements of the CES model. Speaking in terms of LSM as a graph, vertices correspond to $C(s)$ and edges to $S_I(s)$. Input into the network comes from some external states $E(s)$, and their coupling to the network is via $S_E(s)$. A greater challenge is to express the LSM

in dynamical CES terms. To do so requires accurate characterization of each $S(s)$ function above.

Step one is to identify candidate states to make up $x_O$ and $x_E$. For these graphs, however, there is a step zero where we must define at what level we consider the system in question. For instance, a single node may be a system by itself, with all other connecting nodes comprising its environment. Likewise, we may consider a collection of nodes to be the system. Finally, this collection of nodes might contain the entire graph, at which point only the input is part of the environment. To remain the most general, we will treat a collection of nodes as a system. In this way, we may characterize the other choices as the special cases of either a collection of one node or all nodes.

The components of our system are then defined to be some set of nodes $n_O \in N$, where $N$ is the set of all nodes. The environment is, by the CES definition, $n_E = \{n : G_{nx} = 1, x \in n_O\}$, where $G$ is the adjacency matrix of the graph. Provided these sets of nodes, the state vectors $x_O$ and $x_E$ are a partition of $\mathbf{v}$ from Eq. (5.2). That is $x_O = \{v_i : n_i \in n_O\}$ and $x_E = \{v_i : n_i \in n_E\}$.

Structural links, speaking in CES terms, are those linkages that can affect the state of a component. Internal structure, $S_{int}(x_O)$ are all links between elements of $n_O$. We also must consider outgoing links, $S_{ext}^{O \to E}(x_O, x_E)$ and incoming links, $S_{ext}^{E \to O}(x_O, x_E)$. For this particular system, there is only one type of dynamics. As such, all $S(s)$ functions are formally equivalent dynamically. A second issue is that Eq. (5.2) is time-discrete. Fortunately we may write down the Dynamical CES formulation as time-discrete as well.

As a first step, the adjacency matrix $G$ may be reordered with nodes $n_O$ before nodes $n_E$ so that $G$ is a block matrix,

$$G = \begin{pmatrix} G_{OO} & G_{OE} \\ G_{EO} & G_{EE} \end{pmatrix} \tag{5.3}$$

where $G_{XY}$ is an adjacency matrix concerning connections from set $X$ to set $Y$. Provided this ordering, it is then possible to write four versions of Eq. (5.1) with the full $G$ replaced by the four block elements in Eq. (5.3). This reordering and rewriting of the dynamics establishes four systems corresponding to their organism-environment relation, with differing semantics depending on the choice of $n_O$ at the outset.

## 5.6 Delay in a recurrent network

Propagation of states throughout a recurrent network such as LSM reservoirs can be characterized by two processes, recursion delay and dilution. Both are effects of repeated function application. Each time the update function is applied to incoming connections at a particular node, a unit of time is added, and states are mixed together. To make this clear, consider again the small graph in Fig. 5.3 with node $N1$ as the input state. Fig. 5.5 shows the full history of $N2$ at time-step 6. The figure is showing that $N2$ at time 6 is expanded to a function of the input

$$N2(6)$$

$$f(\bar{v_5}, N5(5), N3(5))$$

$$f(\bar{v_4}, N4(4)) \quad f(N4(4), N5(4))$$

$$f(N5(3)) \quad f(\bar{v_3}, N4(3))$$

$$f(\bar{v_2}, N4(2)) \quad f(N5(2))$$

$$f(N5(1)) \quad f(\bar{v_1}, N4(1))$$

Figure 5.5: Expansion of the value of $N2$ at time 6. The input time series is labeled $v_1$ through $v_5$, and its presence is highlighted by the dashed squares. At each time step, the value of a state is a function $f$ of states connected to it.

node, $N5$, and $N3$, all at time 5. The value of $N5$ at time 5 is in turn a function of the input node and N4 at time 4. In this fashion the value of $N2$ at time 6 can be fully expanded.

What Fig 5.5 illustrates is that at any time, here time-step 6, the value of a node contains traces of the full history of the nodes connecting to it. For the example of $N2(6)$, the expansion shows all past values of the input, $N4$ and $N5$. In this example it is also clear that function application is equivalent to time. Expansion by one function application is exactly equivalent to regressing by one time step. Each time the function is applied to one or more nodes, however, the time series of those nodes become diluted. As such, the histories present in the expansion of Fig. 5.5 are transformations of the past.

## 5.7 Discussion

Anticipation through coupling, by definition, requires consideration of both anticipated and anticipating system. For a wide variety of cases, including those presented in Chapters 3 and 4, the systems to be considered are organism and environment. The CES model promotes a relatively unified organism-environment system, where both pieces are necessarily considered together. As such, it appears to be the right system model for anticipation qua anticipating synchronization. As a minimal model of a system, the CES model serves well to outline the ab-

stract relationship between organism and environment, but it lacks appropriate concreteness for direct connection to anticipating synchronization. Particular kinds of recurrent networks, exemplified by the reservoirs used in LSMs, have the same abstract structure, such that a mapping can be made between them. This mapping serves to marry the concrete syntactic nature of anticipating synchronization with the semantics of organism-environment systems. Simulations on LSM reservoirs producing anticipation may be analyzed with respect to dynamical synchronization on the syntactic side, or with respect to organism-environment on the semantic side.

# 6 Discussion

In chapter 2, the delay-coupling arrangement from H. U. Voss (2000) is generalized to accommodate different numbers and types of time-shifted couplings. This generalization helps to define classes of systems based on the type of coupling. A notation labels the classes $U_m S_n$, according to the number ($m$ and $n$) of time-shifted couplings to points in the driver ($U$) or driven ($S$) system. Simulations of these classes using the Rössler-Spring system show particular behaviors for the different classes of couplings.

Empirical results for class $U_0 S_1$ already show agreement with theoretical expectations. Extending these results in line with Chapter 2, Chapters 3 and 4 set up experiments matching other classes, $U_0 S_n$ and $U_\infty S_1$, respectively. Again, the empirical results show agreement with theoretical expectations.

At this point it is prudent to ask what these results mean and what they do not. The point of this work is not to describe a better working model for anticipation. What these results show is that variations in the phenomenology of real anticipatory behavior can be explained by a dynamical system that does not rely on predictions from an inference-based, small scale model. It is not necessarily that the dynamical model works better than others, but that it works at all.

This type of anticipation is described in a way that it can apply to all levels of organism. That is, anticipation at the level of general principle. Given this principle, it is expected that organisms become anticipatory. Anticipation is another way of synchronizing, which is a phenomenon already known to happen opportunistically (Pikovsky, Rosenblum, Kurths, & Hilborn, 2003).

Chapter 3 presents a manual tracking task that differs from the task used in Stepp (2009). Participants are still asked to synchronize with the current state of a master time-series, but instead of receiving feedback at a single delay, feedback is presented at multiple delay times. This change is analogous to changing from class $U_0 S_1$ to $U_0 S_n$ with $n > 1$. Fig. 2.2 shows how the Rössler-Spring system reacts to such a change, which is mirrored qualitatively by participant behavior. Specifically, anticipating synchronization is stable at longer delay times, apparently due to multiple delays, and anticipation makes up for approximately 40-50% of the imposed delay.

Chapter 4, at least in abstraction, is also a deviation from the $U_0 S_1$ class, this time adding coupling to not just the present state of a master time-series, but to a section of upcoming values. This type of coupling is also simulated in Chapter 2 for a Rössler-Spring system, and qualitative features are replicated. The constant shift between gaze and road is predicted by there being a particular look-ahead value that maximizes anticipation. Furthermore, anticipation with coupling to

upcoming values is expected to increase at a greater rate than imposed delay.

The high level question being addressed by this research is "Where does anticipation come from?" Anticipatory behavior clearly exists, from single celled organisms to humans. Why that is true is not nearly so clear. One way to treat the preceding results is as a suggestion that anticipation arises naturally. Extremely simple organisms show anticipatory behavior (Saigusa et al., 2008). If anticipation arises naturally then of course simple organisms will exploit it.

There is more to the suggestion that anticipation arises naturally; why it would do so. One thread that remains constant throughout the previous chapters is that of time-shifted coupling, especially delayed feedback. Events in this universe take time to happen; nothing happens instantly. This fundamental property of the universe has anticipation as a consequence. Anticipation does not always obtain, but support for it is woven into the lowest levels of physical law.

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